

PRINCIPLES AND PERFORMANCE OF KINEMATIC GPS POSITIONING

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1. INTRODUCTION

The Global Positioning System (GPS) is a satellite based all weather navigation system presently being implemented by the United States of America. Its primary purpose is to satisfy the navigation requirements of the U.S. and allied military forces for the decades to come. The GPS navigation signals are available to non-military users as well, at a reduced accuracy level. The basic principle of operation in navigation mode is to measure pseudoranges between a GPS receiver and at least four GPS satellites for the determination of the receiver position and clock offset. Similarly, receiver velocity and clock rate are determined from measurements of range rates to at least four satellites. Continuous visibility of at least four satellites at any place on the surface of the earth is provided by a constellation of 24 GPS satellites in 12 hour orbits. For a more complete description of the GPS system components and operation we refer to Wells et al. (1987).

The specified GPS position accuracy (e.g. DOT & DOD, 1990) is of the order of 10 metres (one sigma) for authorized military users, and of the order of 50 metres (one sigma) for all other users. However, soon after the first satellites were launched into orbit, the potential for a much higher accuracy in differential or relative GPS positioning was realised. Surveyors and geodesists developed procedures for the determination of baseline components between survey markers with accuracies at the part per million (ppm) level. This accuracy improvement is achieved primarily by accumulating measurements over an extended period of time in static mode, and by measuring phases of the GPS carrier signals in addition to pseudoranges. By now, *Static Relative GPS Positioning* or *GPS Surveying* is an accepted procedure with various levels of sophistication for tasks ranging from the establishment of geodetic control networks to monitoring of crustal deformations on a global scale.

For a navigational user, the GPS provides absolute position and velocity information instantaneously, and in real time, at the system design accuracy levels specified above. The terms *Differential GPS Positioning* and *Differential GPS Navigation* are usually reserved to describe procedures, where a GPS monitor station provides error reduction messages to navigational users of GPS. Typically, this error reduction leads to differential position accuracies of a few metres for distances of up to 100 kilometres. We use the terms *Kinematic GPS Positioning* or *Kinematic GPS Surveying* for those procedures, which allow the instantaneous determination of differential positions at the much higher surveying accuracy level of a few ppm. In this context, instantaneous means based on single epoch measurements only. Obviously, this leads to an enormous reduction of observation time compared to conventional static GPS surveying. Moreover, Kinematic GPS Positioning can also determine positions along the trajectory of a continuously moving vehicle. The term Kinematic GPS Positioning is used for both real time and post mission applications.

In this article, we will describe the principles of Kinematic GPS Positioning, starting with a review of the measurements available from a GPS receiver. Then we will examine the effect of errors limiting the performance of Kinematic GPS. Finally, we shall discuss present trends and new developments in various aspects of Kinematic GPS Positioning.

2. GPS MEASUREMENTS

Depending on the particular receiver type, a GPS receiver can measure some or all of the following three distinct quantities by comparing the signal received from the GPS satellites with a similar signal generated within the receiver: The *pseudorange* by comparing code modulations of the received signal with those of the receiver generated signal, the

carrier phase by comparing the carrier signal received from the satellite with the carrier signal generated in the receiver, and the *Doppler measurement* by comparing the frequency of the received carrier signal with the frequency of the receiver generated signal. The code modulations can be resulting from either the C/A-code, or from the P-code; carrier phases can be measured on either one GPS signal frequency, or on both GPS frequencies (cf. Wells et al., 1987).

GPS observation equations relate the measurements to physically meaningful quantities. The GPS measurements pseudorange, P , and carrier phase, Φ , are primarily measures for the satellite to receiver range, and can be represented by

$$P = \rho + d\rho \quad (2.1)$$

$$\Phi = \rho + d\Phi. \quad (2.2)$$

Both, the pseudorange and the carrier phase measurements are in units of length, and

$$\rho = \| \mathbf{r} - \mathbf{R} \| \quad (2.3)$$

is the geometric distance between the satellite antenna position, \mathbf{r} , at signal transmission time (as represented by the satellite ephemerides) and the (unknown) receiver antenna position, \mathbf{R} , at signal reception time (Bold letters are used to identify matrices and vectors). The terms $d\rho$ and $d\Phi$ represent errors and biases in the measurements and consist of

$$d\rho = d\rho + d_{\text{ion}} + d_{\text{trop}} + c \cdot (dt - dT) + \eta_c + \varepsilon_p \quad (2.4)$$

$$d\Phi = d\rho - d_{\text{ion}} + d_{\text{trop}} + c \cdot (dt - dT) + F + \eta + \varepsilon_\Phi \quad (2.5)$$

where

- $d\rho$ is the range error resulting from satellite ephemerides errors,
- d_{ion} is the range error caused by (dispersive) ionospheric signal delay,
- d_{trop} is the range error caused by tropospheric signal delay,
- c is the speed of light in vacuum,
- $c \cdot dt$ is the range error caused by the satellite clock error,
- $c \cdot dT$ is the range error caused by the receiver clock error,
- F is the carrier phase ambiguity,
- η_c is the range error caused by code signal multipath,
- η is the range error caused by carrier signal multipath,

and $\varepsilon_p, \varepsilon_\Phi$ are the range errors in pseudoranges and carrier phases resulting from measurement noise. The carrier phase ambiguity, F , remains constant as long as the receiver keeps phase lock to the incoming signal. Every time the signal is lost and reacquired, the value of F changes by an unknown number of full carrier signal wave lengths.

Most kinematic positioning procedures do not analyse these pseudorange and carrier phase measurements directly. Usually, so-called single differences are formed between the measurements of simultaneously operating GPS receivers.

$$\Delta P = \Delta\rho + \Delta(d\rho + d_{\text{ion}} + d_{\text{trop}} - c \cdot dT + \eta_c) + \Delta\varepsilon_p \quad (2.6)$$

$$\Delta\Phi = \Delta\rho + \Delta(d\rho - d_{\text{ion}} + d_{\text{trop}} - c \cdot dT + \eta) + \Delta F + \Delta\varepsilon_\Phi \quad (2.7)$$

Differencing, indicated by the operator Δ , eliminates the effect of the satellite clock error, and reduces the range errors caused by ephemerides errors and ionospheric delay (Wells et al., 1987). This reduction in error contamination is being paid for by a reduction in geometric information: The single differences of GPS measurements retain strong information about the relative position of the simultaneously observing receivers, but only very weak information about the absolute position of the individual receivers. A further elimination of the receiver clock error is achieved through differencing between simultaneous measurements to different satellites, indicated by the operator ∇ .

$$\nabla\Delta P = \nabla\Delta\rho + \nabla\Delta(d\rho + d_{ion} + d_{trop} + \eta_c) + \nabla\Delta\varepsilon_P \quad (2.8)$$

$$\nabla\Delta\Phi = \nabla\Delta\rho + \nabla\Delta(d\rho - d_{ion} + d_{trop} + \eta) + \nabla\Delta F + \nabla\Delta\varepsilon_\Phi \quad (2.9)$$

$\nabla\Delta$ is often referred to as the double difference operator. The relative position information of the measurements is contained in the double difference of the geometric range, $\nabla\Delta\rho$. Because of positive spatial correlation, the double differences of range errors caused by satellite ephemerides errors, $\nabla\Delta d\rho$, and by ionospheric refraction, $\nabla\Delta d_{ion}$, are greatly reduced compared with the corresponding undifferenced values. On the other hand, the errors that are uncorrelated between satellites and receivers, are increased by a factor of two. Usually, this will be the case for multipath induced errors, and for the measurement noise.

Doppler measurements are obtained by comparing the frequency of the received carrier signal with the frequency of the receiver generated signal. The frequency difference is induced by the relative velocity between transmitter (satellite) and the GPS receiver. As such, it is primarily a measure of the receiver to satellite range rate and is contaminated by the rate of ephemerides and signal propagation errors. From eqn. (2.9) we obtain the following observation equation for the double difference of the Doppler measurement:

$$\nabla\Delta D = \frac{d}{dt} \nabla\Delta\rho + \frac{d}{dt} \nabla\Delta(d\rho - d_{ion} + d_{trop} + \eta) + \nabla\Delta\varepsilon_D \quad (2.10)$$

The information about the relative velocity between the simultaneously observing GPS receivers is contained in the time derivative of the double difference of the geometric range, $\nabla\Delta\rho$. Equations (2.8), (2.9), and (2.10) are the basic non-linear observation equations utilized in kinematic positioning procedures. For receivers observing n satellite signals simultaneously, there are $n-1$ independent double differences of pseudoranges, carrier phases, and Doppler measurements. It should be noted, that mathematical correlations have been introduced through the differencing process between the $n-1$ measurements in each of these three groups.

3. PRINCIPLE OF KINEMATIC GPS POSITIONING

GPS positioning is the determination of the remote receiver position and velocity from GPS pseudorange, carrier phase and Doppler measurements, through equations (2.8) - (2.10). The position and velocity of the master receiver are assumed to be known. For the discussion of the basic principles of Kinematic GPS Positioning, we will omit in this section the measurement errors resulting from ephemerides errors and signal propagation effects; they will be discussed in more detail in section 4. The simplified non-linear observation equations

$$\nabla\Delta P = \nabla\Delta\rho + \nabla\Delta\varepsilon_P \quad (3.1)$$

$$\nabla\Delta\Phi = \nabla\Delta\rho + \nabla\Delta F + \nabla\Delta\varepsilon_\Phi \quad (3.2)$$

$$\nabla\Delta D = \frac{d}{dt} \nabla\Delta\rho + \nabla\Delta\varepsilon_D \quad (3.3)$$

have to be linearised in the usual way with respect to the coordinates and velocities of the remote receiver. Linearisation requires appropriate approximate values for the remote receiver position, \mathbf{R}^* , and velocity, \mathbf{V}^* . Taylor expansion of the satellite to receiver range in equations (3.1) - (3.3) yields:

$$\nabla\Delta P - \nabla\Delta\rho^* = \nabla\Delta\partial\rho/\partial\mathbf{R} |_{\mathbf{R}^*} d\mathbf{R} + \nabla\Delta\varepsilon_P \quad (3.4)$$

$$\nabla\Delta\Phi - \nabla\Delta\rho^* = \nabla\Delta\partial\rho/\partial\mathbf{R} |_{\mathbf{R}^*} d\mathbf{R} + \lambda N + \nabla\Delta\varepsilon_\Phi \quad (3.5)$$

$$\nabla\Delta D - \frac{d}{dt}\nabla\Delta\rho^* = \nabla\Delta\partial\left(\frac{d}{dt}\rho\right)/\partial V |_{R^*,V^*} dV + \nabla\Delta\varepsilon_D \quad (3.6)$$

where

$$dR = R - R^* \quad (3.7)$$

$$dV = V - V^* \quad (3.8)$$

and

$$\nabla\Delta\rho^* = \nabla\Delta \|r - R^*\| \quad (3.9)$$

is the range double difference computed from satellite ephemerides, r , and the approximate receiver position, R^* . Only first order terms have been retained in the above equations, thereby assuming the approximate values for positions and velocities to be so accurate as to render negligible the higher order terms. The nominal range rate double difference $dV \Delta\rho^*/dt$ is calculated as a function of satellite position and velocity (from ephemerides) and the approximate values for receiver position and velocity R^* and V^* , respectively. The partial derivatives of the range rate double difference with respect to the receiver velocity in eqn. (3.6) are identical to the partial derivatives of the range double difference with respect to the receiver coordinates in eqns. (3.4) and (3.5). The double difference of the carrier phase ambiguity term, $\nabla\Delta F$, is an integer number of full carrier wavelengths. Therefore, it has been replaced in equation (3.5) by the product of the carrier signal wavelength, λ , and an (unknown) integer number, N .

If at a particular measurement epoch pseudoranges, carrier phases and instantaneous Doppler are observed to n GPS satellites, then the resulting linearised observation equations for the $n-1$ independent double differences for each observation type can be assembled in linear equation systems as:

$$\ell_P = A dR + \varepsilon_P \quad (3.10)$$

$$\ell_\Phi = A dR + B N + \varepsilon_\Phi \quad (3.11)$$

$$\ell_D = A dV + \varepsilon_D \quad (3.12)$$

The left hand side contains the double differences of measurements minus the range (range rate) double difference computed from approximate values. A is the design matrix for position and velocity, B is the design matrix for the ambiguity terms. Note that A appears in all three equations. Three position increments (dR), three velocity increments (dV), and $n-1$ integer numbers (N) are the primary unknowns in the kinematic positioning problem.

3.1 Kinematic GPS Positioning with predetermined carrier phase ambiguities

A variety of different algorithms for the solution of the GPS observation equations has been described in the literature. The idea of Kinematic GPS Positioning originated with Remondi (1985) who proposed a procedure based on carrier phase measurements only. The principle of this approach is to determine the integer numbers N in eqn. (3.11) before the actual kinematic survey begins. Since the carrier phase ambiguities are constant in time, the integer numbers N will remain at their pre-determined values during the survey. Therefore, the only unknowns for each individual measurement epoch are the three position increments for the remote receiver, dR . Provided that carrier phases to at least four satellites in favourable geometry ($\text{rank}(A) = 3$) are measured, the remote receiver position increment is obtained from a standard least squares solution:

$$\hat{dR} = (A^T C_\Phi^{-1} A)^{-1} A^T C_\Phi^{-1} (\ell_\Phi - B N). \quad (3.13)$$

Here, C_Φ is the carrier phase double difference covariance matrix, and the symbol $\hat{}$ signifies the least squares estimate. Several procedures have been devised to determine the integer numbers N beforehand. Probably the simplest of these

makes use of a previously established precise baseline. Differential GPS carrier phase measurements over such a baseline can be utilised to determine the integer number N instantaneously. Since dR is zero for known coordinates, eqn. (3.5) can be directly inverted to yield

$$N = (\nabla\Delta\Phi - \nabla\Delta\rho^*)/\lambda. \quad (3.14)$$

Because of the neglected noise ϵ_Φ , this equation yields a real number for N instead of an integer value. However, it is often possible to establish the correct integer from its real number estimate. Other methods of determining the integer numbers N are available, e.g. the so-called antenna exchange method (Hofmann-Wellenhof and Remondi, 1988). The Kinematic GPS Positioning procedure with carrier phase measurements alone described above takes full advantage of the integer nature of the carrier phase ambiguities. However, it requires that at least four GPS satellite signals are tracked in phase lock at any time. Failure to do so necessitates a re-determination of the ambiguity numbers N .

3.2 Kinematic GPS Positioning through the combination of pseudoranges and carrier phases

If an initialisation of the carrier phase ambiguity numbers is not possible, the pseudorange and carrier phase measurements can be adjusted simultaneously in a sequential least squares process providing estimates for both the remote receiver position increments dR and the ambiguity numbers N . The simultaneous least squares inversion of eqns. (3.10) and (3.11) for a particular measurement epoch leads to the solution

$$\begin{bmatrix} d\hat{R} \\ \hat{N} \end{bmatrix} = M^{-1} U \quad (3.15)$$

with

$$M = \begin{bmatrix} A^T C_\Phi^{-1} A + A^T C_P^{-1} A & A^T C_\Phi^{-1} B \\ B^T C_\Phi^{-1} A & B^T C_\Phi^{-1} B + C_N^{-1} \end{bmatrix} \quad (3.16)$$

and

$$U = \begin{bmatrix} A^T C_P^{-1} \ell_P + A^T C_\Phi^{-1} \ell_\Phi \\ B^T C_\Phi^{-1} \ell_\Phi + C_N^{-1} \hat{N} \end{bmatrix}. \quad (3.17)$$

C_P denotes the covariance matrix of the pseudorange double differences; \hat{N} and C_N are the least squares estimates for the integer numbers N and their covariance matrix respectively, as obtained in the adjustment of the previous measurement epoch. These terms take care of the fact that the ambiguity numbers do not change between measurement epochs. Initially, the accuracy of the positioning results of this sequential process is dominated by the pseudorange noise. As more and more measurements are accumulated, pseudorange noise is increasingly filtered out. The process can be smoothed backward to improve the initial position accuracy (cf. Kleusberg, 1986; Kleusberg and Georgiadou, 1991). In this procedure, the integer nature of the ambiguity numbers is not implicitly utilised.

3.3 Velocity determination from Doppler measurements

The relative velocity of the remote GPS receiver can be determined directly through least squares inversion of eqn. (3.12) as

$$d\hat{V} = (A^T C_D^{-1} A)^{-1} A^T C_D^{-1} \ell_D. \quad (3.18)$$

Since the design matrix A is position dependant, the velocity determination should be proceeded by a sufficiently accurate positioning procedure. C_D is the covariance matrix of the double differences of the Doppler measurements.

3.4 Position and velocity determination through Kalman filtering

A Kalman filter typically applied in Kinematic GPS Positioning (e.g. Cannon, 1989) combines the simultaneous least squares solution of eqns. (3.10) through (3.12) with a parameter transition model. Loosely speaking, the transition model describes the change of the parameters (positions, velocities, ambiguities) between subsequent measurement epochs. In doing so, the parameter information is carried over from epoch to epoch. Several alternative realisations of a Kalman filter for Kinematic GPS Positioning are described in detail by Schwarz et al. (1989).

3.5 Positioning with carrier phase smoothed pseudoranges

The procedure described in section 3.2 is based on a combination of pseudorange and carrier phase measurements in the positioning process. A different method for the simultaneous use of both measurement types was first discussed by Hatch (1982). He proposed to combine the time series of pseudoranges and carrier phases for each individual satellite separately, in a sequential process. In this method the carrier phases are utilised to filter out the pseudorange noise on a satellite by satellite basis. The resulting "smoothed" pseudoranges are more accurate which is reflected in a corresponding change of the covariance matrix for the pseudorange double differences. The remote receiver positions are obtained from the least squares inversion of eqn. (3.10) according to

$$d\hat{R} = (A^T C_P^{-1} A)^{-1} A^T C_P^{-1} \ell_P \quad (3.19)$$

with ℓ_P and C_P being the smoothed pseudorange double differences and their covariance matrix respectively.

4. KINEMATIC GPS PERFORMANCE AND ACCURACY LIMITATIONS

When discussing the various Kinematic GPS Positioning procedures in the previous section, we neglected the effects of satellite orbit and multipath errors, as well as the tropospheric and the ionospheric delay. We also neglected possible errors in the coordinates of the master station. The effects of most of these errors on *Static Relative GPS Positioning* have been investigated thoroughly (e.g. Santerre, 1990). For long observation sessions, the short period components of the errors are absorbed in the adjustment residuals. Long period components of the errors yield systematic relative positioning errors, at least for a homogeneous satellite distribution. The error propagation is quite different in Kinematic GPS Positioning for two reasons. First, the position determination is primarily based on single epoch measurements and the distinction between short and long period error components becomes meaningless. Second, the number of redundant measurements is rather small. Therefore, the major part of the measurement errors is absorbed by position errors, and a minor part by the adjustment residuals.

The error propagation from measurements into the position estimates is mathematically described by the law of covariance propagation. The simplification of this law into a scalar equation leads to the concept of Position Dilution Of Precision (PDOP). PDOP is the ratio between a scalar position accuracy measure and the scalar measurement accuracy measure and depends on the geometry between satellites and the receiver (Wells et al., 1987). For the full GPS constellation, PDOP will vary between 2 and 4. Presently PDOP values up to 10 are not uncommon during the time windows of four satellite visibility.

The accuracy of Kinematic GPS Positioning depends on how well the different error terms in eqns. (2.8) and (2.9) are modelled. Ephemerides errors, dp , of GPS satellites are typically below 20 m, which leads to differential position

accuracies of about 1 ppm, equivalent to position errors of 10 cm for a distance of 100 km between master and remote station (e.g. Wells et al., 1987).

The ionospheric delay, d_{ion} , can be eliminated if dual frequency measurements are available. For single frequency data, the effect of uncorrected ionospheric delay on differential positions may be of the order of a few ppm, during periods of high solar activity (e.g. Georgiadou and Kleusberg, 1988a). However, local variations in the ionosphere can introduce errors of similar size in addition to these average values. An example for such position errors is given in Figure 4.2. Shown are the differences between height coordinates of an aircraft obtained in Kinematic GPS Positioning with respect to two different stations, using single frequency measurements (Kleusberg and Georgiadou, 1991). The two master stations were separated by 100 km. Analysis of the single frequency data of the two master stations in Static GPS Positioning mode revealed periodic variations in the adjustment residuals up to ± 20 cm, which disappeared in dual frequency data processing. In the Kinematic GPS Positioning of the aircraft with single frequency data, these ± 20 cm differential delay variations are mapped into position errors. For the data shown in Figure 4.2, PDOP values were varying between 3 and 5.

Tropospheric delay, d_{trop} , is usually accounted for by an atmospheric profile model driven by meteorological data (Janes et al. 1991). Tropospheric delay modelling errors are of the order of a few centimetres, which can map into 10-20 centimetres of position errors for large PDOP values.

Carrier signal multipath errors depend on the sensitivity pattern and the environment of the GPS antenna. Typically, these errors show a cyclic behaviour, and repeat from day to day for a static receiver (Georgiadou and Kleusberg, 1988b). Figure 4.1 shows the effect of severe signal multipath errors on the height coordinate obtained in Kinematic GPS Positioning mode. The two curves in the Figure depict the apparent variations in height of a static baseline processed in kinematic mode for two consecutive days. Clearly visible is the repeat nature of the position errors from day to day. The errors are up to ± 20 cm, with a PDOP value equal to 10 during data collection. Multipath errors do not depend on the distance between master and remote station.

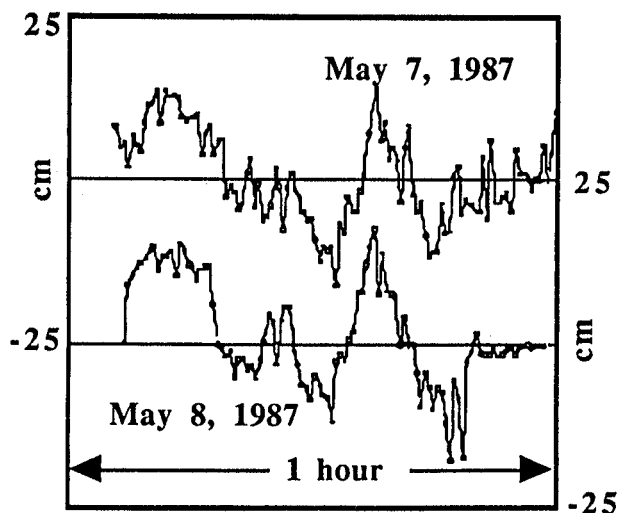


Figure 4.1: Vertical kinematic position errors due to carrier signal multipath

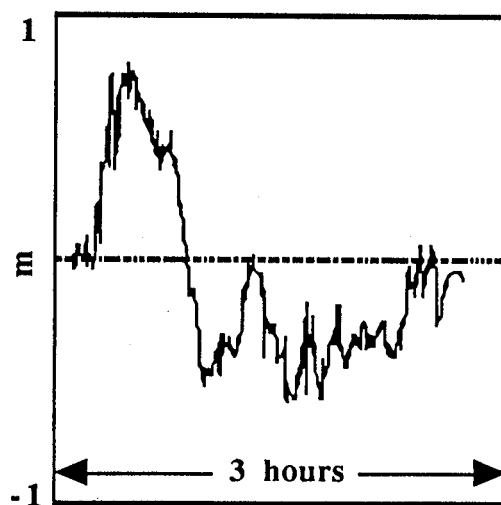


Figure 4.2: Vertical kinematic position errors due to differential ionospheric delay

Another error source to be discussed here is associated with the coordinates of the master station. In section 3 we assumed that the master station coordinates are known exactly. In reality, the station coordinates will be in error to a

lesser or higher degree. These coordinate uncertainties will introduce additional errors. They are described in detail by Santerre, (1990).

The GPS carrier phase measurement noise is of the order of a few millimetres. Even with large PDOP values, it will not contribute more than a few centimetres to the kinematic GPS position errors. It affects GPS-derived positions less than any other error source.

5. TRENDS IN KINEMATIC GPS

The general trend in GPS positioning is characterised by a reduction in hardware cost and size, and by more sophisticated computer software for data analysis. For Kinematic GPS Positioning in particular, a considerable effort is directed towards algorithm and software development for rapid carrier phase ambiguity resolution, i.e. the correct identification of the integer number N in eqn. (3.11).

It was shown in section 3.1, that kinematic positioning based on carrier phase measurements alone is possible, once these integers have been determined. There it was assumed, that the integers are determined in static mode before the actual survey begins, and that at least four satellite signals are tracked in phase lock at any epoch in time during the survey. Both these requirements cannot be met economically in several kinematic GPS applications. Therefore, several schemes for rapid carrier phase ambiguity resolution "on the fly" have been proposed recently in the literature.

Four different approaches to rapid ambiguity resolution can be distinguished. Euler and Goad (1991) combine dual frequency carrier phase and pseudorange double differences for the determination of ambiguity numbers and double differences of the geometric range and the ionospheric delay. They show that with proper weighting of the measurements, ambiguity resolution times of less than a minute can be achieved. The procedure is independent of receiver motion and can be applied in static and kinematic mode.

The other three approaches to rapid ambiguity resolution exploit the integer nature of the carrier phase ambiguities in the position determination process. With the Ambiguity Function Technique, Mader (1990) finds a position solution based only on fractional phase measurements. To do so, all positions within a predetermined 3-D search area have to be tested. The search area can be established through pseudorange positioning and the corresponding position covariance matrix. Depending on the size of the search area, very powerful computers are required for this method.

An approach called "Least Squares Search Technique" was introduced by Hatch (1989). It is similar to the Ambiguity Function Technique in the sense, that the integer nature of the ambiguities is built into the position estimation process, and in the sense that a search over a confidence region has to be performed. However the search is not done in 3-D position space but in integer ambiguity space. This leads to a vast reduction in computer power requirement.

The third approach by Frei and Beutler (1990) utilises the result of an ordinary least squares adjustment for the correct identification of the ambiguities. This adjustment result consists of position estimates, real number estimates of the ambiguities and the associated covariance matrix. The covariance matrix defines a confidence hyper-ellipsoid, within which the most likely integer combination of ambiguities is selected.

Anyone of the positioning procedures outlined in section 3, including the rapid ambiguity resolution techniques described above can be executed in real time, if sufficient data communication is available from the master to the remote station. Various alternative communication software configurations are presently tested for suitability. Structures and formats for data transmission are presently in the process of being defined.

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ABSTRACT

This contribution reviews the basic principles of Kinematic GPS Positioning using various types of GPS measurements and several different positioning algorithms. The limitations in position accuracy resulting from errors in GPS measurements are summarised. Present trends in Kinematic GPS Positioning are outlined.

PRINZIPIEN UND LEISTUNGSFÄHIGKEIT DER KINEMATISCHEN POSITIONSBESTIMMUNG MIT GPS

ZUSAMMENFASSUNG

Dieser Beitrag beschreibt die grundlegenden Prinzipien der kinematischen Positionsbestimmung mit GPS mit verschiedenen GPS Meßgrößen und mit unterschiedlichen Positionierungsalgorithmen. Die Beschränkung der Positionsgenauigkeit resultierend von Fehlern in den GPS Messungen wird zusammengefaßt. Die zur Zeit aktuellen Forschungsrichtungen in der kinematischen Positionsbestimmung mit GPS werden dargestellt.

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