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# AEROTRIANGULATION WITH GPS - METHODS, EXPERIENCE, EXPECTATION

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## 1. INTRODUCTION

Today the satellite based NAVSTAR Global Positioning System (GPS) is already a well-known term in the geodetic and photogrammetric community. The system is, at the moment, still in the build-up phase. Nevertheless GPS is already applied to positioning tasks in practical geodetic work. At present, there are 16 GPS-satellites in operation : 6 Block I satellites, 9 Block II satellites and 1 Block IIa satellite. The Block I satellites were especially developed for experimental testing of the system performance. The Block II satellites constitute the operational space segment of GPS. The present satellite constellation provides coverage for 2-dimensional positioning 24 hours a day. Three-dimensional positioning is possible daily for a period of 4.5 hours with 5 - 7 satellites and for two periods of 1 hour with 4 - 6 satellites (elevation cut-off angle 10 deg, PDOP[1] < 5). These observation periods move backward in time from day to day by 4 minutes (2 hours per month). The situation improves by each new launch of a GPS-satellite. According to the present state of information, 24 hour 3-dimensional positioning is scheduled for the end of 1993 (GPS BULLETIN 3/90).

In aerial photogrammetry attention is concentrated on the utilization of GPS for photo flight navigation and for determining the positions of the air survey camera at the individual moments of exposure by relative kinematic positioning. These GPS positions can be introduced in the combined block adjustment as additional observations. The prime effect is the substantial reduction of necessary terrestrial control points, which is of great practical and economic importance, since the establishment of ground control by terrestrial survey is time-consuming and often constitutes the major financial costs in aerial triangulation projects.

At present a number of tests and pilot applications in GPS-supported aerial triangulation are carried out in certain countries, but they are still under study due to the considerable expenditure of time involved in establishing absolute control. This paper therefore can only summarize the present experience in airborne kinematic GPS positioning gained in several tests. It also projects the impact of combined block adjustment on the accuracy of blocks and the reduction of ground control, as can be expected on the basis of computer simulations and theoretical propagation of errors. Practical recommendations are given for the operational execution of GPS-supported aerial triangulation, and how practical conditions can be taken into account.

## 2. EXPERIENCE IN AIRBORNE KINEMATIC CAMERA POSITIONING

(1) From the operational point of view, three practical features have to be mentioned, concerning the precise positioning by GPS of an aerial camera at the individual moments of exposure: The GPS receiver and the aerial camera operate independently of each other in time, and the GPS antenna and the aerial camera are separated spatially in the aircraft. In case of processing carrier phase observations the problem of the phase ambiguity arises in addition. These three items do not constitute serious problems. They can be treated in practice as follows :

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[1] PDOP = Position Dilution of Precision = $\sqrt{Q_{xx} + Q_{yy} + Q_{zz}}$

(1.1) Modern air survey cameras are able to generate an impulse at the midpoint of the exposures (ZÜGGE 1989). These impulses can be recorded on the GPS receiver time scale by means of a special photogrammetric input device available in several GPS receivers. The positions of the GPS aircraft antenna can then be computed by interpolation techniques. It has been shown empirically, that a linear interpolation between the 2 neighbor positions is sufficient, if the observation rate of the GPS receiver is ≤ 0.6 sec.

(1.2) The eccentricity between the GPS antenna and the aerial camera is to be measured with respect to the camera coordinate system by terrestrial surveying techniques (e.g. SCHWIERTZ/DORRER 1991). It will be considered in the combined block adjustment and is, with regard to the GPS positioning, not of particular interest. With regard to the combined block adjustment, some additional remarks have to be made.

If the camera is operated in locked-down mode, the components of the eccentricity vector are constant in the camera coordinate system. However, the camera is usually not locked-down to the aircraft body during a flight mission. It is tilted or at least rotated to compensate the crab angle of the aircraft with regard to the planned flight line. Consequently, the GPS antenna eccentricity is variable within the camera coordinate system. Two solutions are conceivable. If the GPS antenna could be mounted vertically above the aerial camera, so that the horizontal components of the eccentricity are zero or at least close to zero, the effects of camera rotation onto the eccentricity can be neglected in most aerial triangulation tasks. For applications with high accuracy requirements (e.g. photogrammetric point determination) or in case the GPS antenna could not be mounted close to the camera, the actual alignment of the camera with regard to an aircraft fixed coordinate system has to be measured directly through a special device. The eccentricity parameters, measured with respect to the camera coordinate system at a reference epoch, can then be corrected in a preprocessing stage for the variation of the camera alignment with respect to an aircraft fixed coordinate system. The input for the combined block adjustment consists either of only one eccentricity parameter set or of a list of eccentricity parameters for the individual times of exposure.

(1.3) The interferometric GPS phase observations refer to the last incoming cycle of the carrier wave. The total number of integer cycles between satellite and receiver is unknown. After the first phase observation the GPS-receivers count the number of incoming cycles. As long as the counting is not interrupted, only the initial number of cycles of the first measurement is unknown. This initial ambiguity has to be determined before the actual kinematic positioning, i.e before the aircraft starts moving. For computing the ambiguity parameters the starting position of the aircraft with respect to the reference point has to be known. Alternatively it can be determined from stationary GPS observations prior to the actual flight. As will be shown the determination of the initial phase ambiguity may be abandoned altogether, in certain cases.

(2) The high accuracy potential of airborne GPS positioning, especially on the basis of carrier phase observations, has been empirically proven by several independent investigations of photogrammetrically controlled test flights (VAN DER VEGT 1989, HEIN 1989, DORRER/SCHWIERTZ 1990, FRIESS 1990). The main results summarized in the following were obtained from the test flight "Flevoland", which still seems to be, up to now, one of the most comprehensively investigated test flights. They refer to relative kinematic GPS positioning, with one stationary receiver on a known point on the ground and one receiver in the aircraft, and to postprocessing of L1 carrier wave phase observations by single differences.

- The internal accuracy of the observations was empirically determined to $\sigma_\Phi$ = 1.4 mm for carrier phase measurements and to $\sigma_P$ = 1.2 m for C/A-code pseudoranges (without S.A.[2]). The theoretically expected internal precision of positioning is thus 0.7 mm and 5.7 m respectively, if a satellite constellation geometry with a PDOP of 5 is assumed.

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[2] S.A = Selective Availability, Degradation of the C/A-Code Signal and Broadcast Ephemeris

- The empirical noise analysis of the derived position coordinates resulted in a precision of approx. 2 cm for carrier phase processing and in a precision of approx. 7 m for the differential pseudorange solution.
- The analysis of the external accuracy of airborne kinematic positioning is based on the comparison of the GPS-positions with the camera positions, independently determined by photogrammetric block adjustment. From those analyses two main results are obtained: GPS airborne kinematic positioning is subject to residual systematic errors, which lead to a time-dependent variation (drift) of the coordinates. The drift seems to be linear in first approximation, at least for time intervals of up to 15 min. After applying linear corrections an accuracy level of a few centimeter is achieved.

The internal precision of the GPS-observations gained by empirical data analyses is slightly better than theoretically expected (1% of the wavelength). It should be noticed that the data used refer to a certain GPS receiver (Sercel TR5S-B) and to a period of time when Selective Availability was turned off. The quoted empirical internal precision of the coordinates is not in total agreement with the theoretical expectation, but is at least very close. However, the established internal precision of airborne GPS camera positioning is far better than the accuracy required for most photogrammetric applications.

The empirical investigations have confirmed, beside the high internal precision, that the derived GPS aircraft positions can be affected by drift errors. The occurring drift can be attributed to remaining uncertainties in the a priori corrections (e.g. atmospheric refraction) and to unmodelled error effects (e.g. satellite orbit errors), in spite of applying differencing techniques (relative positioning). The sensitivity of kinematic positioning with respect to an uncertainty of the initial ambiguity parameter has to be mentioned in particular. Special analyses have shown, that an error in the order of 1 dm in the initial baseline, used for computing the initial ambiguities, already causes a noticable drift in the subsequent kinematic positioning. Whether, and to which extent, drift errors are acceptable depends on the intended use of GPS data and on the possibility of subsequent correction. It is important to notice, in the context, that linear GPS drift errors can be corrected in combination with aerial triangulation. If the drift errors can be taken into account in this way, the accuracy of the GPS positions can reach a level of a few centimeters.

(3) With regard to the operational conditions of photogrammetric flight missions certain practical difficulties must be considered. It cannot be presumed, that photo flights start always from the same airport. Therefore the starting position of the aircraft with respect to a reference station cannot be assumed to be known in advance. In most cases the position will have to be determined before take-off by GPS baseline determination. This however requires stationary observations recorded over a period of 1 hour, which is, from the practical point of view, not really acceptable, especially with respect to the momentary still limited observation period, and to prevailing airport conditions.

At most airports, the pilots have to follow the instructions of the tower for take-off. It will in general not be possible to start with a low take-off angle, which would be necessary for avoiding interruptions of the satellites signals. Also during turns shadowing of individual satellites cannot be avoided. Consequently, it cannot be guaranteed that the GPS signal reception is continuous in airborne applications, i.e without any interruption. Signal interruption of individual satellites dissolves the previous ambiguity solution, which has to be redetermined. For P-Code dual frequency receivers this does not constitute a particular problem; algorithms for ambiguity determination in airborne applications are described e.g. in SEEBER/WÜBENNA 1989. With C/A-code single frequency receivers the problem can also be solved in principle, if the receiver has a multi-channel design and the capability of tracking all visible satellites, so that redundant observations are obtained. Then it should be possible to observe at least 4 satellites continuously, which enables continuous positioning based on which the ambiguity parameters of the interrupted observations can be re-assessed. It is also possible to handle situations where only 3 satellites or less continue to be observed whilst all other satellite signals are interrupted over short intervals of time (< 5 sec). Using prediction methods for the receiver clock error and/or for the aircraft trajectory the ambiguity parameters can be solved for in such cases. There appear, however, two kinds of difficulties: The constellation geometry of the non-shadowed satellites is poor and does not allow airborne positioning with decimeter accuracy, which would be necessary for precise ambiguity determination of interrupted signals. Consequently, new drift errors can arise caused by inaccurate ambiguity parameters. If less than 4 satellites are observed, for a short time interval, the positions of the aircraft antenna have to be predicted. The potential of predicting the trajectory of an aircraft over several seconds of time within a certain accuracy is limited, especially during take-off and during turns. But it is exactly in such situations that the signal interruptions are likely to happen, due to satellite shadowing by the wings of the aircraft.

It must also be considered that a photo flight can take up to 5 hours of time or more. The distance between the aircraft and the stationary GPS receiver on the ground may amount to several hundred km. Also a flight mission may combine several smaller projects quite some distance apart. It would not be economical to place a reference receiver in every area. As the reduction of unmodelled error effects of the GPS observations by differencing techniques decreases with time and distance systematic GPS positioning errors will increase. Therefore drift errors are to be expected to occur normally, which will have to be handled accordingly during the combined block adjustment.

## 3. RECOMMENDATIONS FOR AIRBORNE CAMERA POSITIONING

In view of the operational conditions of photogrammetric flight missions it is recommended for GPS-supported aerial triangulation, to evaluate the GPS data independently for each photo strip, instead of considering one total flight mission as one processing unit. The phase ambiguities can, in this case, be computed approximately via a differential pseudorange solution for the first position of each strip. This method implies, however, the occurrence of drift errors. Such drift errors are acceptable as they can be corrected during the combined block adjustment. It is emphasized, that stripwise processing of GPS data is only recommended in connection with aerial triangulation i.e. with combined block adjustment.

The method of treating flight strips seperately has certain advantages from the operational point of view. First of all, the precise initial baseline for the ambiguity determination is no longer required, the data recording can start at the beginning of the first photo strip and, moreover, the flight maneuvers during take-off and during the turns can be carried out as usual, without paying attention to satellites. Stripwise processing of GPS-data in connection with combined block adjustment has already been applied to the data of the test flight Flevoland. The results have shown that with stripwise linear drift corrections the high accuracy level of the GPS camera positioning can be maintained and that the method of stripwise GPS positioning can be applied in principle. Further investigations are necessary however to verify how far drift errors can be approximated by linear correction terms.

## 4. IMPACT ON COMBINED BLOCK ADJUSTMENT

(1) In application to aerial triangulation it is assumed that the positions of the GPS aircraft antenna are interpolated onto the individual times of camera exposure. The interpolated GPS antenna coordinates are introduced into the combined block adjustment as additional observations for each camera position via appropriate observation equations. These observation equations include linear drift parameters[3] for each coordinate as

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[3] $a_{oi} + a_{1i} (t_j - t_0)$, $a_{oi}$, $a_{1i}$ = unknown drift parameter, i = X,Y,Z, $t_j$ = time of exposure, $t_0$ = reference time

additional unknowns. They approximate and correct the drift errors of the GPS antenna positions in the combined block adjustment. Linear drift corrections also imply and can be interpreted as additional datum transformations. The drift modelling has to be flexible. Depending on the case the drift parameters may be chosen stripwise or common for several strips. If the GPS observations are continuous for a complete photo-flight, without any interruption, one set of parameters ($a_{oX}...a_{1Z}$) may be sufficient for a complete block. If stripwise GPS data processing is to be applied, as recommended above, one set of drift parameter has to be introduced for each of the individual photo strips. The mathematical model of GPS-supported aerial triangulation has been described and discussed in several papers (Colomina 1989, Ackermann 1990, Friess 1990a).

(2) The unkown drift parameters require some additional consideration. Their determinability must be guaranteed by a corresponding ground control configuration and/or flight pattern. For GPS supported aerial triangulation the photogrammetric block is assumed to be geometrically determined in the conventional sense, i.e. with standard overlap and standard tie-point distribution. Two different ground control configurations are recommended : (a) 4 XYZ ground control points, located more or less in the corners of the block, and a chain of vertical control at either front end of the block; (b) 4 XYZ ground control points and, instead of additional vertical control points, two cross strips. With either configuration the linear drift parameters can be safely determined, and the block adjustment can be carried out. The suggested use of 4 ground control points is considered necessary for solving the datum problem. Aerial triangulation without any ground control is possible in principle. It is normally not acceptable however, as the results refer to the WGS 84 rectangular coordinate system.

(3) In Ackermann 1991 the accuracy of adjusted GPS-supported photogrammetric blocks is analyzed theoretically, based on the inversion of the normal equation matrix. These studies have shown, that the accuracy properties of the adjusted GPS blocks are extremely favourable. The GPS camera positions control a block very well, suppressing essentially any propagation of errors. The accuracy of GPS blocks depends (within reasonable limits) very little on ground control, on block size, and on GPS accuracy. However, the accuracy depends markedly on wether and how many drift parameters are applied in the adjustment, as they weaken the geometry of the block noticeably. The resulting block accuracy, expressed as r.m.s values $\mu_{X,Y}$ and $\mu_Z$ of the horizontal and vertical standard deviations, respectively, of all adjusted tie-points, can be summarized in the following simple relationships:

| | | $\mu_{X,Y}$ | $\mu_Z$ |
|---|---|---|---|
| no drift parameter (1) | : | $1.0\ \sigma_o\ s$ | $1.5\ \sigma_o\ s$ 1) |
| one set of drift parameter for the complete block (2) | : | $1.7\ \sigma_o\ s$ | $1.7\ \sigma_o\ s$ 1) |
| one set of drift parameter for each individual strip (3) | : | $2.1\ \sigma_o\ s$ | $2.3\ \sigma_o\ s$ 2) |
| case (3) with 2 cross strips | : | $1.5\ \sigma_o\ s$ | $2.0\ \sigma_o\ s$ 3) |

$\sigma_o$ : precision of the image coordinates
s : photo-scale

1) 4 XYZ ground control points
2) 4 XYZ ground control points + 2 chains of vertical control
3) 4 XYZ ground control points + 2 cross strips

The above theoretical results are directly valid for a block of 6 strips with 21 photographs and combined bundle block adjustment. The precision of the ground control point coordinates and of the image coordinates of the ground control points has been assumed to be equal to $\sigma_o$ and $\sigma_o \cdot s$, respectively. The accuracy of the GPS camera positions is assumed to be $\sigma_{GPS} \leq \sigma_o \cdot s$ i.e. to correspond to the photogrammetric measuring accuracy. The combined block adjustment can be done equally well with the independent model method. The accuracy results are expected to be very similar to the bundle method.

The above results are not restricted to the particular assumptions which have been made. The block size has only a slight influence. According to ACKERMANN 1991, for smaller blocks the accuracy deteriorates within 10% for larger blocks the accuracy improves within 10%. The most important result of these studies is the fact, that with decreasing accuracy of the GPS camera positions ($\sigma_{GPS} \geq \sigma_o \cdot s$) the block accuracy deteriorates with a much slower rate than $\sigma_{GPS}$ itself. If, for example, the accuracy of the GPS positions amount to $\sigma_{GPS} = 10 \cdot \sigma_o \cdot s$, the above values for the block accuracy $\mu_{X,Y}$ and $\mu_Z$ decrease only by the factor 2. Taking into account that the required block accuracy for medium and small scale mapping is only in the order of $2 \cdot \sigma_o \cdot s$ to $5 \cdot \sigma_o \cdot s$, there exists a considerable margin with respect to the required accuracy of the GPS camera positions. It means that the requirements for the GPS positioning accuracy are not stringent at all, in case of aerial triangulation for mapping purposes.

In ACKERMANN 1991, the results of these accuracy analyses are taken as a basis for deriving the required accuracy for the GPS camera positions to meet given specifications of the block accuracy $\mu_{X,Y}$ and $\mu_Z$. It can generally be stated, that GPS-supported aerial triangulation with the specified minimum of ground control can meet the accuracy demands for the complete spectrum of photogrammetric mapping tasks, even if drift parameters per strip have to be applied (with the possible exception of very large scale applications).

## 5. CONCLUSIONS

It can be stated in conclusion that GPS-supported aerial triangulation is ready for practical application. Software for GPS kinmematic positioning as well as software for the combined block adjustment (PAT-B, PAT-M) are available. Several geodetic GPS receivers are on the market. If the recommended ground control configurations in addition to stripwise linear modelling of GPS drift errors in the combined block adjustment are applied, kinematic relative camera positioning for aerial triangulation is a highly operational, robust and most economic method which can change thoroughly aerial photogrammetry within a short time.

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## ABSTRACT

The satellite based navigation system NAVSTAR GPS permits the eccentric determination of the position of the aerial camera at the individual moments of exposure. These positions introduced in the combined block adjustment as additional obervations of the camera positions, enable an aerial triangulation with a minimum of terrestrial control points.

The accuracy behavior of airborne kinematic GPS positioning is comprehensively presented, based on empirical analyses. According to present practical experiences in airborne kinematic GPS positioning, recommendations are given for the operational application of a GPS-supported aerial triangulation. The theoretical accuracy behavior of combined block adjustment is demonstrated.

# AEROTRIANGULATION MIT GPS - METHODEN, ERFAHRUNGEN, ERWARTUNGEN

## ZUSAMMENFASSUNG

Das satellitengestützte Navigationssystem NAVSTAR GPS erlaubt die exzentrische Bestimmung der Position der Luftbildkammer zu den Aufnahmezeitpunkten. Diese Positionen, eingeführt in eine kombinierte Blockausgleichung als exzentrische Beobachtungen der Kamerapositionen, gestatten eine Aerotriangulation mit einem Minimum an terrestrischen Paßpunkten.

Die Genauigkeitseigenschaften der kinematischen Positionsbestimmung mit GPS im Flugzeug werden auf der Grundlage empirischer Untersuchungen zusamengefaßt dargstellt. Anhand der bisherigen praktischen Erfahrungen mit kinematischer GPS Positionsbestimmung im Flugzeug werden Empfehlungen für die operationelle Durchführung einer GPS-gestützten Aerotriangulation gegeben. Die theoretischen Genauigkeitseigenschaften der kombinierten Blockausgleichung werden aufgezeigt.

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