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## 1. INTRODUCTION

The fusion reactor JET -abbreviation Joint European Torus- at Culham, UK, has taken up operations and it is working to the satisfaction of its designers, as far as we know. Whether energy output exceeds already energy input, we do not know. Figure 1 shows a model of the reactor.

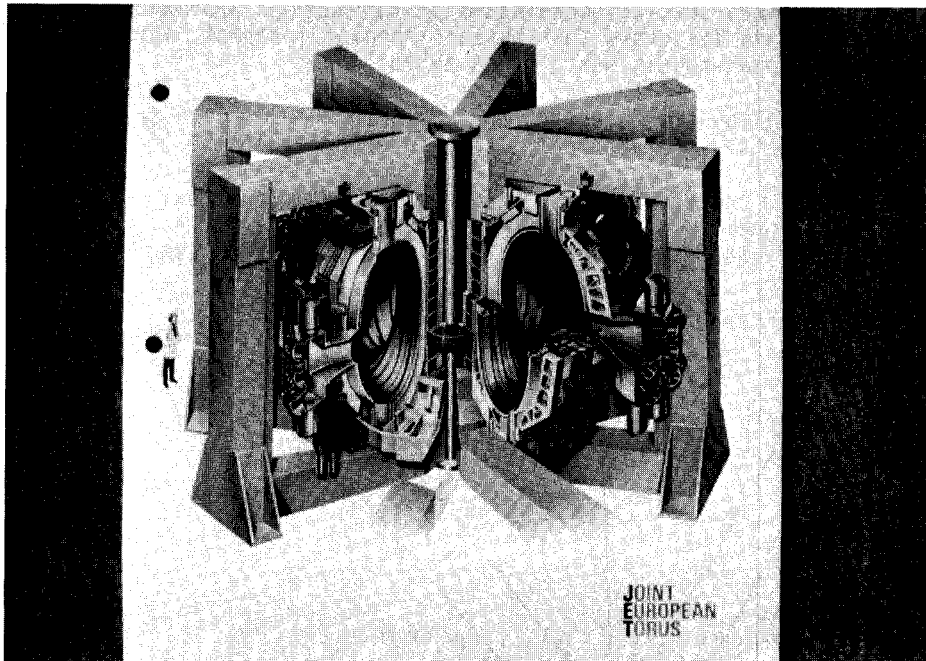


Fig. 1: Model of the fusion reactor JET (taken from /7/)

The aim of this large scale, multi-national research project is the controlled fusion of hydrogen nuclei. The project was planned and financed by several European countries.

JET is a fusion reactor of the TOKOMAK type. In principle it is a ring shaped chamber in which hydrogen is exposed to high acceleration under very high temperatures. The chamber is isolated by high vacuum. Toroidal and poloidal coils exert magnetic forces that prevent collision of the hydrogen nuclei with the chamber's stainless steel wall. The components of the torus, such as the magnet core, the toroidal and poloidal field coils, the vacuum vessels, the mechanical structure, etc. were manufactured by industrial plants in different European countries. They were assembled at Culham first to eight -theoretically identical- segments, named octants, and then mounted to make up the torus. Large elements of the octants, such as the vacuum vessels, were exposed to loading and heating tests prior to the assembly. Since each octant had to be interchangeable with a ninth, a stand-by octant, it was necessary to document the final shape of the vacuum vessels and the assembled octants.

## 2. DOCUMENTATION OF GEOMETRIC DIMENSIONS OF THE TORUS' COMPONENTS

The main interest of the physicists and mechanical engineers was to obtain a record of angular relationships between mounting faces, distances between mounting points and the deviations of mounting faces from a true plane. Close-range photogrammetry was accepted as a tool for documentation and measurement.

The task was in particular:

- A precise photogrammetric form-control of one toroidal magnetic-field coil. The coil was photographed at the BBC plant in Mannheim and restituted in Stuttgart.
- Photogrammetric documentation of all vacuum vessels and octants. The photography was performed by the Culham photographic laboratory with a calibrated camera UMK 10/1318.
- Exemplary photogrammetric restitution of selected vacuum vessels and assembled octants in Stuttgart.

The photogrammetric documentation had to be organized in such a way that items of interest could be measured at any time prior or after the final mounting of the torus. The arrangement of camera positions and the marking of points on the objects with self-adhesive targets was identical for all components (Fig. 3).

The photogrammetrically determined spatial coordinates of the targets provided the data for the calculation of planes, angles and distances that were of special interest. Information for the absolute orientation of the stereo-models was defined as follows:

- The plane of symmetry between mounting faces "A" and "E" (Fig. 2) - derived from spatial coordinates of selected targets- is a vertical plane, independent of the component's position in the assembly hall.
- The distance of the geometric center of this plane from the rotational axis of the torus is the designed (theoretical) distance.
- The stereo-models are scaled via measurements on calibrated level rods imaged in the photographs (Fig. 3).

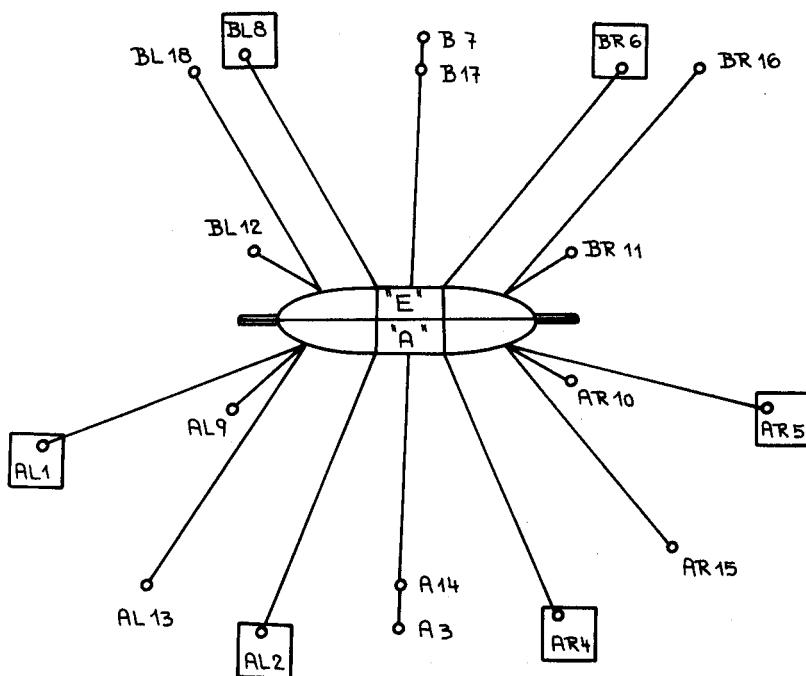


Fig. 2: Camera positions for stereo-coverage of a vacuum vessel

With these definitions a comparison of measured and designed dimensions was possible. Exemplary results are given in the table below.

$\varphi'$  = calculated angle;  $\varphi$  = theoretical angle;  $d\varphi = \varphi - \varphi'$

plane 1	plane 2	$\varphi'$	$\varphi$	$d\varphi$
XZ-plane	joint A	22.43	22.50	+0.07
XZ-plane	joint E	22.52	22.50	-0.02
XZ-plane	1/4 SA	11.20	11.25	+0.05
XZ-plane	1/4 SE	11.34	11.25	-0.09
joint A	joint E	44.95	45.00	+0.05
joint A	side A	2.53	2.50	-0.03
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joint E	side E	2.46	2.50	+0.04
	mech.struc.			
top port	bottom port	0.08	0.00	-0.08
top A	top E	157.56	157.50	-0.06

Table 1: Comparison of calculated and designed angles between selected mounting faces

For the analysis of the data the method applied was that of independent stereo-models, which were connected via targets and scaling rods common to at least 2 models. It was a sequential process and the accuracy of the results was directly dependent on the measurements on joints "A" and "E", resp. on the orientation of the adjusting planes calculated with all spatial points derived from those measurements. The computations were very time-consuming. This fact motivated the development of the program BITRI to handle convergent close-range photographs in a global bundle adjustment.

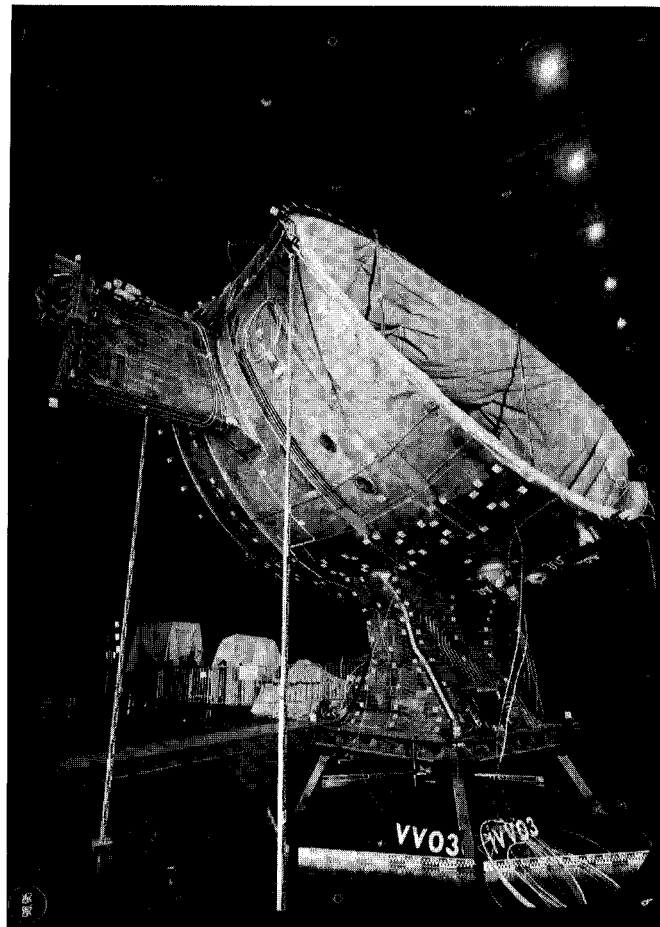


Fig. 3: Vacuum vessel prepared for photogrammetric documentation

### 3. ADJUSTMENT OF BUNDLES OF CONVERGENT EXPOSURES

In order to analyse all image coordinates and all terrestrial data in one global adjustment and to compute the coordinates of the points of the object, a proper mathematical approach was chosen. This approach allows for the calculation of approximate coordinates of the points of the object at the same time. The approach, first published in /5/, performs the calculation of approximate coordinates and the final adjustment and analysis of the photogrammetric measurements in two steps of a calculational process. Both steps are based on proper models of adjustment and were incorporated in the computer program BITRI. The basic idea of the strategy is to include all data right from the beginning into a simultaneous calculational process. In addition this has the advantage that the information with respect to terrestrial control points is minimal and only necessary to fix a proper datum of the object, i.e. 7 parameters are sufficient and no further fixed positions are required.

The calculation of approximate coordinates and of the bundle adjustment following is based on a common mathematical model. Similar to the linear model of adjustment using similarity transformations in the plane, the approximate coordinates for the adjustment of convergent exposures are derived from a suitably chosen set of linear equations. To formulate equations for the adjustment of convergent exposures being linear in the coordinates of the points, some additional information with respect to a rough orientation of the exposures will be needed. These data must only be known to approximately 20 degrees and may easily be taken from a sketch of the arrangement of the planned camera positions or it might be derived from registered azimuth and inclination data of the camera.

#### 3.1 THE MATHEMATICAL APPROACH

The starting point of the analysis is the functional model of the spatial similarity transformation - collinearity equations- :

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = m_{ik} \cdot R_k \begin{bmatrix} x_{ik} - x_{ok} \\ y_{ik} - y_{ok} \\ -c_k \end{bmatrix} + \begin{bmatrix} X_{ok} \\ Y_{ok} \\ Z_{ok} \end{bmatrix} \quad (1)$$

$X_i, Y_i, Z_i$  are the coordinates of any object point  $i$  being observed in the exposure  $k$  with the image coordinates  $x_{ik}, y_{ik}$ .  $X_{ok}, Y_{ok}, Z_{ok}$  are the coordinates of the centre of projection,  $R_k$  is the matrix of rotation and  $m_{ik}$  is the individual scale value for each image point. In order to be most general with respect to the rotations of the convergent exposures, an orthogonal rotation matrix based on quaternions according to /9/ was chosen. Instead of three independent parameters  $\alpha, \beta, \gamma$  there are four parameters  $r_{1k}, r_{2k}, r_{3k}, r_{4k}$  which are functional dependent with each other. The functional dependency is described by:

$$r_{1k}^2 + r_{2k}^2 + r_{3k}^2 + r_{4k}^2 = 2 \quad (2)$$

#### 3.2 LINEAR SUBSTITUTE MODEL FOR THE CALCULATION OF APPROXIMATE COORDINATES

Equation (1) might be regarded as a system of error equations for the parameters, the object points  $X_i, Y_i, Z_i$ , the centers of projection  $X_{ok}, Y_{ok}, Z_{ok}$  and the scale parameters  $m_{ik}$ . If the rotation matrix and the inner orientation of each exposure is roughly known in advance, this system is linear and can be solved for the parameters according to the rules of least squares adjustment.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{ik} = m_{ik} \cdot R_{ik} \begin{bmatrix} x - x_o \\ y - y_o \\ -c \end{bmatrix}_{ik} + \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix}_k - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i \quad (3)$$

In addition, if the calculation is regarded as an iterative process, the parameters of the rotation matrix may also be solved for after the first iteration following system (3).

Instead of choosing  $m_{ik}$  in (3) as a parameter, the scale factor of the rotation matrix, namely  $2/(r_{1k} + r_{2k} + r_{3k} + r_{4k})$  and  $m_{ik}$  will be combined to a new parameter  $\bar{m}$ . Then R in system (3) is represented as follows:

$$\begin{bmatrix} v_{x_{ik}} \\ v_{y_{ik}} \\ v_{z_{ik}} \end{bmatrix} = \bar{m}_{ik} \begin{bmatrix} r_{1k}^2 + \frac{r_{2k}^2 + r_{3k}^2 + r_{4k}^2}{2} & r_{1k}r_{2k} - r_{3k}r_{4k} & r_{1k}r_{3k} + r_{2k}r_{4k} \\ r_{1k}r_{2k} + r_{3k}r_{4k} & r_{2k}^2 + \frac{r_{1k}^2 + r_{3k}^2 + r_{4k}^2}{2} & r_{2k}r_{3k} - r_{1k}r_{4k} \\ r_{1k}r_{3k} - r_{2k}r_{4k} & r_{2k}r_{3k} + r_{1k}r_{4k} & \frac{r_{1k}^2 + r_{2k}^2 + r_{4k}^2}{2} + r_{3k}^2 \end{bmatrix} \begin{bmatrix} x_{ik} - x_0 \\ y_{ik} - y_0 \\ -c \end{bmatrix} + \begin{bmatrix} X_{0k} \\ Y_{0k} \\ Z_{0k} \end{bmatrix} - \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} \quad (4)$$

The system is nonlinear with respect to the rotation parameters. The nonlinearity is only of a third order polynomial type. Thus the iterative calculational process based on the repeated linearization of (4) and the adjustment of the linearized error equations will show a fast convergence and will be nearly independent from the the quality of the approximate orientation. Because of the parameters  $\bar{m}$  being only present in the three equations of each point they can easily be eliminated from the system of linearized error equations using SCHREIBER'S method of elimination.

The approach described above includes all observations right from the beginning into the calculation. Thus blunders can be detected using the strategy of adjustment applying a method of down weighting and the method of data snooping and taking advantage of the convergence behaviour of the approach.

#### 4. THE BITRI APPROACH FOR BUNDLE ADJUSTMENT OF CONVERGENT EXPOSURES

In order to regard the image coordinates  $x_{ik}, y_{ik}$  of the object points as stochastic parameters which have to be treated according to the rules of least squares adjustment, the approach (1) will be transformed into:

$$\begin{bmatrix} x_{ik} \\ y_{ik} \\ 0 \end{bmatrix} = m_{ik}^{-1} R_k^T \begin{bmatrix} X_i - X_{0k} \\ Y_i - Y_{0k} \\ Z_i - Z_{0k} \end{bmatrix} + \begin{bmatrix} x_{0k} \\ y_{0k} \\ c_k \end{bmatrix} \quad (5)$$

Here the property of orthogonality of the rotation matrix R is observed. The unknown parameters of the least squares adjustment may be subdivided into 4 groups:

- the orientation parameters of the individual exposures - the rotation parameters  $r_{1k}, r_{2k}, r_{3k}, r_{4k}$  and the coordinates of the projection centers;
- the coordinates of the object points  $X_i, Y_i, Z_i$ ;
- the parameters of the inner orientation of the camera;
- the scale parameters  $\bar{m}_{ik}$ .

On the basis of the described formulae, including the elimination of  $\bar{m}_{ik}$  in the linearized equations, the FORTRAN program BITRI was developed /5/, /2/. In addition to the observations "image coordinates" further types of observations may be included into the adjustment process. These types are i.g.:

- distance observations;
- individual coordinates of object points;
- parameters as observations.

Using the approach described above all observations can be analysed applying statistical tests according to /1/. The proper weighting of the observations can be evaluated during the adjustment applying variance component estimation according to /3/. The modular structure of the program system follows the strategy of the adjustment package OPTUN for terrestrial observations /4/ and includes the setting up and the automatical optimization of the profile structure of the system of normal equations and the use of the profile during solution and inversion. Thus a large number of parameters may be solved for in a very efficient way.

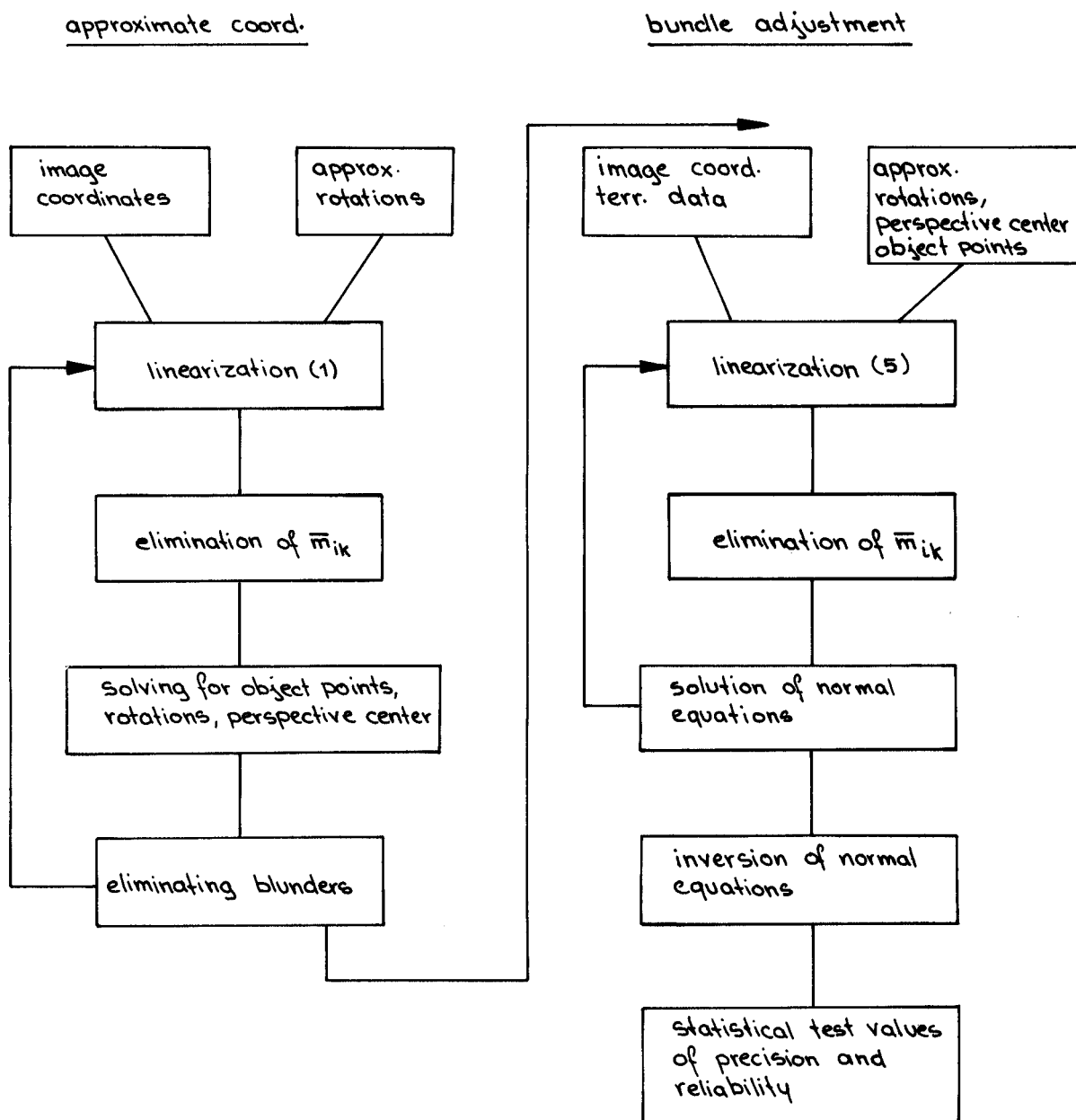


Table 2: Flow-chart of program BITRI

5. PRACTICAL EXAMPLE FOR THE APPLICATION OF BITRI

Figure 2 shows the arrangement of the camera positions for the coverage of a vacuum vessel. Six exposures and 15 points common to several exposures were used in the calculations with BITRI by Bühler /2/. Only one iteration was necessary for the initial value calculation with the orientation of the cameras taken from fig. 3 and the inclination of the optical axes was assumed to be zero. The final adjustment needed just 6 inner iterations.

The result of the adjustment confirmed the good results already achieved by /8/. The advantage of the approach BITRI however is the easy to handle way of analysing the data which is not restricted to a given sequence or to a preselected subset of data. In addition it provides information on the accuracy and the reliability of the result based on a global adjustment. Due to the modular structure of the program system and the strategy of data management being realized, a further evaluation of the object data can easily be performed afterwards, i.e. by the use of the system LOCAL /6/ for deformation analysis. LOCAL allows for a general geometrical comparison of the result with the design model, taking into account the stochastic properties of the coordinates of the object points.

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#### ABSTRACT

The paper deals with close-range photogrammetry as a tool to document the shape of large components of the research-fusion-reactor JET. It shows the analysis of data by the method of restituting independent stereo-models. It also presents a solution for handling convergent close-range photographs in a general bundle adjustment approach.

#### FERTIGUNGSKONTROLLE UND FORMBESTIMMUNG EINES FUSIONSREAKTORS

#### ZUSAMMENFASSUNG

Die Anwendung der Nahbereichsphotogrammetrie zur Dokumentation großer Bauteile wird am Beispiel des Forschungs-Fusionsreaktors JET erläutert. Zunächst wird die Analyse der Daten durch die Auswertung unabhängiger Stereomodelle behandelt und dann eine Lösung zur geschlossenen Bündelausgleichung für konvergente Aufnahmen im Nahbereich angegeben.

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