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HIGH PRECISION DIGITAL IMAGE CORRELATION

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1. Introduction

This is an intermediate report on some development concerning digital image correlation which took place at the Stuttgart Institute of Photogrammetry during the past 3 years. Digital or analog image correlation is nothing new at all, it has seen vast development in remote sensing and in on-line image correlators. Here, the special limited intention has been to test and develop the technique with regard to high metrical precision and for application within the range of conventional photogrammetry. Although we had no previous experience in the field and did, at the outset, not expect any particular breakthrough, the results obtained so far permit optimistic and far reaching conclusions.

The development was initiated by a proposal for digital point transfer submitted to the German Research Foundation within a special research program for remote sensing (Deutsche Forschungsgemeinschaft, Schwerpunkt Fernerkundung). Digital point transfer is of interest to photogrammetry (aerial triangulation) and to remote sensing (matching points of different multispectral images for rectification purposes).

The special scope of the project implied a number of initial decisions, partly intended to make the data handling somewhat easier:

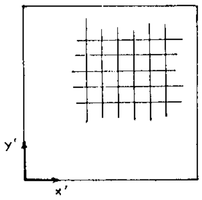
- The considerations were restricted to small digital image areas only.
- The problem of initial approximate matching of homologous image areas was excluded, for the time being. Its solution depends very much on mode and conditions under which the digitisation of images takes place.
- The problems of measuring and calibrating digital image data were also excluded although we had to battle through the difficulties of obtaining data from a microdensitometer in suitable form.
- The investigation concentrated on the geometrical aspects of determining parallax in x and y direction, with potential application in numerical relative orientation, digital elevation models and point transfer (for aerial triangulation and for rectification and matching of multispectral images).
- On the other hand it was decided to correlate local image areas two-dimensionally, i.e. not to refer to epipolar rays.

2. Special Approach

For digital image correlation it is assumed that two homologous image windows of a pair of overlapping photographs are digitized (by a microdensitometer, for instance), resulting in two gray-value matrices G_1 and G_2 (see fig. 1). The pixel arrays define the local coordinate systems. The image contents of the two windows are supposed to overlap sufficiently, i.e. to cover about the same object area.

The task of image correlation is then to transform, say, G_2 onto G_1 in such a way that a best match is obtained, or more specifically, that a certain correlation function is optimized.

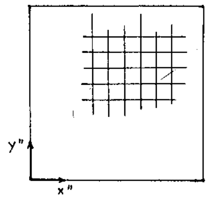
There are several such correlation functions available which might be applied. The most common are maximisation of the corrrelation coefficient or minimisation of the (root mean square value of the) remaining differences of gray values. In either case the cross-products or the differences of gray-values of nominally related pixels are used.



left window (1)

gray-value matrix

$G_1 = G_1(x'_i, y'_i) = G_1(x_i, y_i)$



right window (2)

gray-value matrix

$G_2 = G_2(x''_i, y''_i) = G_2(x_i, y_i)$

Fig. 1: Digitisation of small image areas

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The two gray-value matrices are not identical, for radiometric and for geometrical reasons.

Causes for radiometric differences are:

- Radiation from an object depends on illumination and on spatial direction
- degradation by atmosphere (and camera)
- differences in film processing
- possible differences in microdensitometer measurement

Causes for geometrical differences are:

- different camera orientation, perspective image distortion
- relief displacement
- different image distortion

For high precision image correlation such effects have to be compensated for, to a sufficient degree.

The commonly applied procedures of image correlation obtain the correlation by a systematic trial and error procedure. One window is systematically shifted, until for instance maximum correlation is obtained.

The definite shift values, which represent the local parallax x and y (related to the coordinate systems used) are obtained finally by interpolation. This procedure operates well; it is, however, not too satisfactory from a methodical point of view.

We therefore chose a different approach which seemingly has not been used elsewhere to any extent. The method was conceived and developed independently. We found only later, that it had been mentioned in literature previously. The characteristic features of the approach are, that the transformation parameters are introduced as unknowns which are determined directly by a least squares solution. This approach allows to extend the number of parameters sufficiently in order to compensate for radiometric and geometrical differences of the two windows. In connection with this approach it was advantageous to obtain image correlation by minimizing the gray-value differences rather than maximizing the correlation-coefficient.

2.1 Principle of approach (1-dimensional)

The basic concept of the approach may be explained for the one-dimensional case. The gray-values of the (linear) pixel arrays from the left hand and the right hand image windows represent (discrete) gray-value functions g (x') and g (x"), referring to the coordinate systems x' and x", respectively.

If both coordinate systems, which are sufficiently identical, are superimposed (and relabeled, $x' \rightarrow x'' \rightarrow x$) we obtain $g_1(x)$ and $g_2(x)$ with regard to nominally the same reference system.

Let us ideally assume that both functions are equal, except for a shift x_o of g_2 against g_1, for a scale factor a of g_2 against g_1, and for perturbation by noise, $n_1(x_i)$ and $n_2(x_i)$.

We then obtain for the observed functions:

$$\begin{aligned} \bar{g}_1(x_i) &= g_1(x_i) + n_1(x_i) \\ g_2(x_i) &= g_2(x_i) + n_2(x_i) \\ &= g_1(ax_i - x_o) + n_2(x_i) \end{aligned} \qquad (2.1\text{-}1)$$

Consider now the gray-value-differences $\Delta g(x_i)$ of nominally corresponding pixels i :

$$\begin{aligned} \Delta g(x_i) &= \bar{g}_2(x_i) - \bar{g}_1(x_i) \\ &= g_2(x_i) + n_2(x_i) - g_1(x_i) - n_1(x_i) \\ &= g_1(ax_i - x_o) - g_1(x_i) + n_2(x_i) - n_1(x_i) \end{aligned} \qquad (2.1\text{-}2)$$

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Now, $g_1(ax_i - x_o)$ is linearized with regard to x_o and to a , expanding from the approximation $x_o = 0$ and $a_o = 1$:

$$g_1(ax_i - x_o) = g_1(x_i) - \frac{\partial g_1(x_i)}{\partial x} x_o + \frac{\partial g_1(x_i)}{\partial x} x_i \cdot \Delta a$$

$$= g_1(x_i) - \dot{g}_1(x_i) \cdot x_o + \dot{g}_1(x_i) \cdot x_i \cdot \Delta a \qquad (2.1\text{-}3)$$

If, in addition, the symbol $v(x_i)$ is substituted for the difference of the noise components:

$$v(x_i) = n_1(x_i) - n_2(x_i) \qquad (2.1\text{-}4)$$

equation (2.1-2) can be rewritten:

$$\boxed{\Delta g(x_i) + v(x_i) = -\dot{g}_1(x_i) \cdot x_o + x_i \dot{g}_1(x_i) \cdot \Delta a} \qquad (2.1\text{-}5)$$

Here, $\Delta g(x_i)$ is the difference of the observed gray-values. $\dot{g}_1(x_i)$ is the gradient of the unknown function $g_1(x_i)$. It can, however, be approximated by the gradient of the available, observed function $\bar{g}_1(x_i)$, possibly in combination with appropriate filtering. x_o is the unknown shift parameter of g_2 against g_1, Δa represents the unknown scale factor of g_2 against g_1. Both unknowns are supposed to be differentially small.

Equation (5) has the form of linearized observational equations, with the observations $\Delta g(x_i)$ and the unknowns x_o and $\Delta a = a-1$. They can be solved directly by least squares techniques for the unknowns x_o and Δa, thus minizing

$$\Sigma\, v^2(x_i) = \Sigma\, (n_2(x_i) - n_1(x_i))^2 \qquad (2.1\text{-}6)$$

The solution for x_o and Δa will correspond to the minimum of equation (6); it will in particular represent the shift (parallax) of scaled g_2 against g_1. Of course, any deviation of the basic assumption $g_2(x_i) = g_1(ax_i - x_o)$ will also go into $v(x_i)$.

As for each pair of corresponding pixels one equation (5) is obtained, the least squares solution for x_o and Δa will be highly redundant.

The basic equation (5) is linearized, starting from the approximation $g_1(x_i) = g_2(x_i)$.

As this is, normally, a poor initial approximation, the solution has to be iterated, by relinearisation after each step. This implies, unfortunately, interpolation of the gray-values into the transformed coordinate system (on redefined pixels) which is known as resampling (see below).

2.2 Generalisation of two dimensions

We now generalize the described concept of approach to the two-dimensional case.

The observed discrete gray-value functions $\bar{g}_1(x_i, y_j)$ and $\bar{g}_2(x_i, y_j)$ are given. $\bar{g}_1$ represents the actually unknown function g_1 disturbed by noise $n_1(x_i, y_j)$. $\bar{g}_2$ is assumed to be related to g_1 by

- a geometrical transformation (distortion) T_G , and
- a radiometric transformation (distortion) T_R ,
- and is perturbed by noise.

We use a simplified notation

$$g_1(x_i, y_j) = g_1(z_i)$$

$$g_2(x_i, y_j) = g_2(z_i) \qquad (2.2\text{-}1)$$

Considering first the geometrical transformation T_G of the coordinate system

$$u_i = T_G(z_i, a_m) , \qquad \underline{m} = 1 \ldots \underline{m}$$

giving

$$g_1'(u) = g_1'(z_i, a_m) = g_1(T_G(z_i, a_m)) \qquad (2.2\text{-}2)$$

and then the radiometric transformation T_R

$$g_1''(z_i, a_m, h_k) = T_R(g_1'(z_i, a_m), h_k) = T_R(g_1(T_G(z_i, a_m)), h_k) \quad ; \quad k = 1...k, \; m = 1...m \qquad (2.2\text{-}3)$$

According to the basic assumption, g_1'' is sufficiently identical with g_2:

$$g_1''(z_i, a_m, h_k) \equiv g_2(z_i) \qquad (2.2\text{-}4)$$

If noise $n_2(z_i)$ is added to g_1 directly, for sake of simplicity (it might also be added to g_1' or g_2'', as the assumptions about its origin might dictate), we obtain for the difference Δg of the observed functions $\bar{g}_1$ and $\bar{g}_2$:

$$\begin{aligned}\Delta g(z_i) &= \bar{g}_2(z_i) - \bar{g}_1(z_i) \\ &= g_2(z_i) - g_1(z_i) + n_2(z_i) - n_1(z_i) \\ &= g_1''(z_i, a_m, h_k) - g_1(z_i) + n_2(z_i) - n_1(z_i) \\ &= T_R(g_1(T_G(z_i, a_m)), h_k) - g_1(z_i) + n_2(z_i) - n_1(z_i) \qquad (2.2\text{-}5)\end{aligned}$$

This relationship includes the unknown transformation parameters a_m and h_k. Replacing the noise difference by $v(z_i)$

$$v(z_i) = n_1(z_i) - n_2(z_i) \;, \quad \text{and}$$

expanding equation (2.2-5) to first order terms, starting from approximate values a_m^o and h_k^o, we obtain

$$\Delta g(z_i) + v(z_i) = g_1''(z_i, a_m^o, h_k^o) + \sum_m \left(\frac{\partial g_1''}{\partial a_m}\right)_o \cdot da_m + \sum_k \left(\frac{\partial g_1}{\partial h_k}\right)_o \cdot dh_k - g_1(z_i) \qquad (2.2\text{-}6)$$

If the expansion starts from zero values such that

$$g_1''(z_i, a_m, h_k) = g_1(z_i)$$

then equation (2.2-6) simplifies to

$$\boxed{\Delta g(z_i) + v(z_i) = \sum_m \left(\frac{\partial g_1''}{\partial a_m}\right)_o da_m + \sum_k \left(\frac{\partial g_1''}{\partial h_k}\right)_o dh_k} \qquad (2.2\text{-}7)$$

Equation (2.2-7) represents a linearized observation equation for each pair of related pixels. The "observations" Δg are given weight 1, and the solution for the unknown (increments of the) transformation parameters a_m and h_k can be obtained with the standard least squares techniques.

The partial derivatives, serving as coefficients of differential increments of the unknown parameters, are approximated from the available function $\bar{g}''$ and its gradient functions, or filtered modifications of them. Redundancy of the system is high, as the normal equations are of the order (m+k) only.

Remark:
Instead of equation (2.2-5) an alternate approach is possible, by not using directly the difference $\Delta g = \bar{g}_2 - \bar{g}_1$, but rather the difference $\Delta g = \bar{g}_2 - \bar{g}_1''$, where $\bar{g}_1''$ is the result of a preliminary transformation $\bar{g}_1''(z_i, a_m^o, h_k^o) = T_R(\bar{g}_1(T_G(z_i, a_m^o)), h_k^o)$ with the approximate transformation parameters a_m^o and h_m^o. This approach is actually used in the procedure, as it has advantages in the iteration procedure.

2.3 Resampling

The basic equation (2.2-7) is linearized, starting from possibly poor approximate values for the unknown transformation parameters. The solution has therefore to be obtained by iterative steps. This implies resampling, as after each transformation step a new set of pixels has to be created.

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Originally we applied a sophisticated interpolation method for interpolation of gray-values referring to a new grid of pixels. The gray value at an arbitrary point (x,y) was obtained by a linear combination of covariance functions relating to a limited number (4 or 12) of surrounding pixels. It was found, however, that a bilinear interpolation from the surrounding 4 pixel values is sufficient. This procedure is now used; it simplifies and shortens the numerical computations of resampling considerably.

Remark: The above description only sketches the leading idea of the approach. It is no description of the actual computer program, in which the iterations are interfaced with resampling and with filtering. Also, the actual procedure has been made more symmetrical such that correlation of window 2 into window 1 gives the same result as correlation in the reversed direction.

2.4 Transformations

Equation (2.2-7) has been kept general with regard to the geometrical and radiometric transformations, the parameters of which constitute the unknowns of the problem. They are to be solved directly, in various iteration steps.

Considering that we deal with small windows only (e.g. 32 x 32 pixels of 20 µm x 20 µm each cover an image area of 0.4 mm^2) the geometrical transformations need not go to higher degree terms. For the time being only 4- and 6-parameter linear transformations have been programmed (similarity and affine transformation). This is quite sufficient for locally smooth ground areas, the relief displacement being the only effective cause of geometrical distortion. Higher degree transformations will be investigated.

The radiometric adaption has so far been restricted to the transformation

$$g_i'' = g_i' h_1 + h_2 \tag{2.4-1}$$

The 2 parameters h_1 and h_2 correct for zero-level shift and overall scale difference between the gray-values of both windows. For the task at hand there is no apparent need to apply more versatile transformations.

2.5 Accuracy criteria and definition of parallax

After each iteration step, and after applying the transformation and resampling, the correlation coefficient is calculated, from the products of gray-values of corresponding pairs of pixels:

$$\rho_{g_1 g_2} = \frac{\sum_i g_1(z_i) \cdot g_2(z_i)}{\sqrt{\sum_i g_1(z_i)^2 \cdot \sum_i g_2(z_i)^2}} \tag{2.5-1}$$

Here, the values of g_1 and g_2 are reduced to the average gray-values of each window as zero reference. The correlation coefficient is an indicator for the level resp. the quality of image correlation, which has been obtained. It is not used here as convergence criterion. ρ can reach values as high as 0.99.

Accuracy criteria for the result are obtained from the least squares adjustment in the usual way:

We have first the estimate of the variance factor σ_0^2:

$$\hat{\sigma}_0^2 = \frac{\sum_i (v(z_i))^2}{n-u} = \frac{\sum_i (\overline{\Delta g}(z_i))^2}{n-u} \tag{2.5-2}$$

with n = number of pixels involved, and
u = number of unknown transformation parameters

$\overline{\Delta g}$ represents the differences between the transformed gray-values of the correlated windows remaining after the last iteration step. Thus $\hat{\sigma}_0$ is a truely empirical indicator for the "goodness of match".

With $\hat{\sigma}_0$ the standard errors of the unknown transformation parameters (or of functions of them) can be derived; of which especially the geometrical parameters are of interest:

$$\hat{\sigma}_{a_m} = \hat{\sigma}_0 \sqrt{Q_{a_m a_m}} \tag{2.5-3}$$

where $Q_{a_m a_m}$ is obtained from the inversion of the (small) system of normal equations. The standard deviations $\hat{\sigma}_{a_m}$ are, as such, theoretical values, being

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obtained by propagation of errors. In particular the cofactors $Q_{a_m a_m}$, refer to the assumed mathematical model of the adjustment. However, in the intended application the main and sensitive contribution to $\hat{\sigma}_{a_m}$ is given by $\hat{\sigma}_0$ which is an empirical value, as explained above. Hence it can be concluded, that the estimated values $\hat{\sigma}_{a_m}$ are quite realistic ones in our case.

With the aid of the weight coefficient matrix Q_{ah} the precision of any transformed point can be obtained by applying the rule of propagation of errors.

Finally the question remains of defining a representative image point within a window. The described image correlation of window to window is, in fact, a correlation of areas. However, the purpose of the procedure is to determine parallaxes or to identify homologous points for point transfer. Thus, in the reference window a certain central point must be defined which is represented by the local image area. For this purpose we define a central point x_s, y_s, by the weighted average of all contributing pixels:

$$x_s = \frac{\sum_i x_i \cdot (\dot{g}(z_i))^2}{\sum_i (\dot{g}(z_i))^2}, \quad y_s = \frac{\sum_i y_i \cdot (\dot{g}(z_i))^2}{\sum_i (\dot{g}(z_i))^2} \qquad (2.5\text{-}4)$$

Its corresponding point in the transformed window is obtained by applying the transformation parameters. In (2.5-4) the squares of the gradients $\dot{g}$ serve as weights. The standard errors of the coordinates of the transformed point are obtained, as described above, by propagation of errors. With each image correlation the values for ρ, $\hat{\sigma}_0$, $\hat{\sigma}_s$ are printed out, thus allowing a quality assessment of the actual correlation.

The coordinates x_s, y_s still refer to the coordinate system of the window in question. In order to be able to establish image coordinates, the window-coordinate system must be related to the image coordinate system which is defined by the fiducial marks of the respective image.

Before proceeding further it is duely acknowledged that several collaborators have contributed to the development and the further investigation of the system, in particular E. Wild, R. Koller, A. Pertl and W. Förstner.

3. Results

The described two-dimensional approach was programmed (on the Harris H 100, a medium capacity 24 bit minicomputer) and a number of investigations were carried out. The data for a limited number of samples were originally obtained from digitisation of selected windows on aerial photographs with a Joyce Loebl microdensitometer. The data originated from different aerial photographs, ranging in scale between 1:9000 and 1:28000.

The main experiments are briefly recalled.

3.1 Smoothing and convergence of iterations

It was soon confirmed that the convergence of iterations depends on the degree of smoothing applied to the gray-values. In order to obtain convergence at all and to speed it up strong smoothing is required, especially at the beginning of the process and in case of poor initial approximation. At the end of the process smoothing is abandoned, in order to obtain unbiased results of high precision correlation. At present, the method of finite elements is applied for smoothing.

Table 1 shows one typical example of the effect of smoothing on convergence. As can be seen, about 6-10 iterations were originally required in order to reach convergence. In this particular example the initial off-set of the second window was 6 pixels in x- and 15 pixels in y-direction.

Under the operational conditions which will be described in section 4 the windows match initially within displacements of only 2 to 3 pixels. In this case the latest version of the program requires about 3 to 4 iterations only. In this case no or very little smoothing is necessary. If the initial approximation is very poor, however, the method of phase correlation could be applied in order to obtain quickly improved approximation. This will probably be the only smoothing procedure retained in future program versions.

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Iteration	Original		moderate smoothing		strong smoothing	
	ρ	$\hat{\sigma}_o$	ρ	$\hat{\sigma}_o$	ρ	$\hat{\sigma}_o$
1	0.131	185.8	0.169	166.4	0.247	147.6
2	0.145	184,0	0.263	158.2	0.544	120.3
3	0.151	124.5	0.414	144.9	0.828	84.1
4	0.283	121.8	0.568	129.7	0.982	46.6
5	0.408	115.3	0.666	119.2	0.987	40.4
6	0.458	110.7	0.694	115.1	0.987	40.4
7	0.506	106.9	0.714	111.4	0.995	10.8
8	0.554	102.9	0.729	77.2	0.995	10.6
9	0.575	100.8	0.763	73.1	0.995	10.6
10	0.578	100.1	0.830	71.3	0.995	10.6

Table 1 : Example of the effects of smoothing on the convergence of iterations ($\hat{\sigma}_o$ values refer to a range of 1000 units of gray-values of the microdensitometer)

3.2 Comparison of transformations

It has been described that, so far, two different geometrical transformations have been programmed (4 parameter similarity transformation and 6 parameter affine transformation), in order to correct for geometrical incompatibility or distortion of the two windows. Theoretically the 6 parameter transformation should result in more precise image correlation. The samples of table 2 show, however, that in areas of flat ground the improvement is not always very significant. Nevertheless, the affine transformation is normally applied, as a safeguard.

similarity transformation						
Sample	window size (pixels)	pixel size (µm)	ρ	$\hat{\sigma}_o$	$\hat{\sigma}_p$ (PU)	$\hat{\sigma}_p$ (µm)
road crossing	64 x 64	50	0.976	27.4	0.013	0.7
embankment	70 x 70	20	0.903	49.7	0.028	0.6
field corner I	70 x 70	20	0.976	24.5	0.023	0.5
field corner II	70 x 70	20	0.949	21.6	0.043	0.9
garden	70 x 70	20	0.782	33.6	0.079	1.6
affine transformation						
road crossing	dto.	dto.	0.983	23.4	0.011	0.6
embankment			0.912	49.2	0.028	0.6
field corner I			0.976	24.0	0.023	0.5
field corner II			0.950	22.3	0.058	1.2
garden			0.897	24.3	0.043	0.9

Table 2 : Comparison of 4 parameter and 6 parameter geometrical transformation (σ_p = precision of point transfer, point error; $\hat{\sigma}_o$ refers to units of gray values, PU = units of pixel size)

3.3 Effects of window size and pixel size

In a series of experiments the effects of window size and of pixel size on the precision of the resulting correlation (and on computing time) were investigated. Originally a window size of 64 x 64 pixels or more was used. When reducing window size to 32 x 32 pixels and 16 x 16 pixels, the $\hat{\sigma}_o$ values usually are not much changed. The standard errors of parallax, however, increase, according to reduced redundancy. Nevertheless the obtained parallax accuracy is very high, σ_p increasing from 1 µm and better to 1-2 µm. Table 3 shows the effect of reduced window size. It refers to the case of similarity transformation and is to be compared with table 2.

Increasing pixel size on the other hand is equivalent to some smoothing. Therefore the results were predictable from the smoothing experiments. Table 4 shows

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the results of large pixel size for the same samples as contained in tables 2 and 3. Pixel size smaller than 20 μm will not improve results because the limit of resolution of the aerial photograph would be reached.

sample	window size (pixels)	pixel size (μm)	ρ	$\hat{\sigma}_o$	$\hat{\sigma}_p$ |PU|	$\hat{\sigma}_p$ |μm|
1	32 x 32	50	0.990	19.7	0.026	1.3
2	35 x 35	20	0.886	55.6	0.066	1.3
3	35 x 35	20	0.758	21.8	0.097	1.9
4	35 x 35	20	0.964	19.0	0.045	0.9
5	35 x 35	20	0.701	22.1	0.081	1.6

Table 3 : Effects of reduction of window size

sample	window size (pixels)	pixel size (μm)	ρ	$\hat{\sigma}_o$	$\hat{\sigma}_p$ |PU|	$\hat{\sigma}_p$ |μm|
1	32 x 32	100	0.990	17.2	0.013	1.3
2	35 x 35	40	0.941	38.9	0.040	1.6
3	35 x 35	40	0.987	17.9	0.045	1.8
4	35 x 35	40	0.967	17.9	0.087	3.5
5	35 x 35	40	0.916	21.9	0.065	2.6

Table 4 : Effects of increasing pixel size

3.4 Reversed transformation

The prodecure of image correlation, as described in section 2, is not symmetrical, as one window is transformed onto the other and the gradients are taken from one window only. It has to be expected, therefore, that by reversing the direction of transformation a different result will be obtained. This has been confirmed by experiments. When correlating first window 1 to window 2 and then window 2 back to window 1 the resulting coordinate differences of the central point transferred forth and back have been observed to range from 0.2 μm to 1.5 μm. Although these magnitudes are not very large, the method has been changed to become almost symmetrical, by taking both windows into account when determining the gradients in equation (2.2-7). It would be possible to make the method completely symmetrical.

3.5 Results

Above all the details of the various investigations (which served partly for choosing the strategy of computation) the overall result of the experiments has been, that image correlation is capable of very high precision. Depending on image texture the precision of parallax determination or of point transfer reaches the order of 0.5 μm to 1.0 μm in the image or 0.01 to 0.06 pixel size. This result had not quite been expected. It has been established and confirmed sufficiently, however, to rely upon for black and white aerial photographs. It might be noted, that conventional stereoscopic parallax measurement does not reach this level of precision. Therefore, it is quite difficult, for the time being, to check independently the results of digital image correlation.

It has been mentioned that the programming of the method has been going through several stages of development. With the first version the computing time required was quite long. With the Harris computer H 100 (24 bit minicomputer) it took 80 sec per iteration (with windows of 64 x 64 pixels). By reprogramming this has now been brought down to 2.5 sec per iteration, for 32 x 32 pixels. With the Hewlett Packard computer HP 1000 (16 bit minicomputer, available here with the analytical plotter Planicomp C 100) about the same computing time is obtained. As now 3 to 4 iterations are sufficient, provided the initial approximation is within 2 to 3 pixels, the total computing time for completing the image correlation amounts to about 7 to 10 sec, including smoothing and resampling. It is hoped, that a still further reduction will be possible. Nevertheless, the speed reached now is sufficient to continue extended experiments.

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3.6 Theoretical investigation

Parallel with the experiments theoretical investigations took place in order to establish the theoretical potential of the method and its dependence on various factors. [1] It is not the scope of this paper to delve into theory, but the essential findings can be summarized. The main result is, that the experimentally established high precision of image correlation has been theoretically confirmed.

The precision of image correlation is dependent on a number of factors. The essential ones are:

- image texture (gradients of gray-value functions)
- pixel size
- window size
- preprocessing (filtering)
- signal to noise ratio (SNR)

The essential results are:

a) The empirical results are confirmed by theory, both for high and low contrast image areas. With well textured image areas in high quality aerial photographs the precision of correlation is $\leqq 1$ µm.

The coordinate precision is given by

$$\sigma_x = \frac{1}{\sqrt{n}} \cdot \frac{1}{(SNR)} \cdot \frac{\sigma_g}{\sigma_{\dot{g}}} \qquad (3.6\text{-}1)$$

n = number of pixels, (SNR) = signal to noise ratio (about 5 in aerial photographs)
σ_g = standard deviation of gray-values (after filtering)
$\sigma_{\dot{g}}$ = standard deviation of gradient-values

b) An optimum pixel size exists (20 - 30 µm in aerial photographs) which depends on the image texture (gradients of the gray-value functions), and assuming white noise.

c) Related to the optimum pixel size an optimum precision can be obtained, which depends on the gradient function, on window size and on the signal to noise ratio. The precision reaches values from 1/180 to 1/10 pixel size for high to low contrast aerial photographs. In case of extreme low contrast the standard deviations can go up to 1/2 pixel size or more. Under good conditions on the other hand, the standard deviation of coordinate transfer can be as small or even less than 1% of pixel size.

d) Smaller than optimum pixel size does not improve the precision of correlation.

e) Linear filters do not substantially improve the results.

f) The theoretical analysis seems to confirm the observed rate of convergence of iterations and the procedures applied (smoothing) for speeding up convergence.

The theoretical investigations, making use of the theory of signal processing, will have to be continued. At the moment the optimum window size and automatic selection of best filters are of special interest.

4. Modification of the Zeiss Planicomp for on-line digital image correlation

The surprisingly good results of the digital image correlation made it feasible and desirable to make the method operational for direct practical application in connection with photogrammetric equipment. The initial operation with microdensitometer digitisation and off-line processing was too time consuming and had no direct connection with other photogrammetric work. It was therefore not very suited for direct application from a system's point of view.

It was therefore decided to equip the analytical plotter Zeiss Planicomp C 100 with digital array sensors and to directly process the data on-line on the HP 1000 minicomputer of the instrument. The system had to be arranged in such a way that the stereo-observation and all ordinary functions of the instrument were not disturbed.

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[1] *see W. Förstner: On the geometric precision of digital correlation, Proceedings ISPRS Commission III Symposium, Helsinki, 1982*

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With valuable assistance from the Zeiss company the Planicomp was equipped with 2 Hamamatsu solid state video cameras C 1000 - 35 M (particularly selected, amongst other features, for its high geometrical stability). Each video camera has a sensor array of 244 x 320 MOS (metal oxicide semiconductor) elements. With pixel size 27 µm x 27 µm the array has an extension of 8,8 mm x 6,6 mm. Scanning of the full field takes 4.2 seconds. The sensors are sensitive within a wavelength range of 400 to 1100 nm. The resolution range covers 256 steps (8 bits).

The image area is projected through an auxiliary optical system onto the sensor array. This system has an enlargement factor of 1,35. Hence the effective pixel size, in the image plane, is 20 µm x 20 µm, and the effective window size on the image is about 6,4 mm x 4,9 mm.

The energy is taken from the ordinary illumination system of the instrument. After passing through the diapositive, and before reaching the measuring mark, the light rays meet a beam splitter which diverts about 15 % of the light through the auxiliary optical system on to the sensor array (see fig. 2). 85 % of the light go through the optical path as before. In this way the ordinary stereo observation of the images is not hampered. In fact, the operator does not directly notice the simultaneous digitisation of the window areas in the central part of the field of view.

The output of the video camera is an analog signal, representing the array signals sequentially, column by column. The signal goes through an A/D converter and an interface, to be delivered digitally to the internal memory of the HP 1000 computer of the Planicomp (see fig. 2).

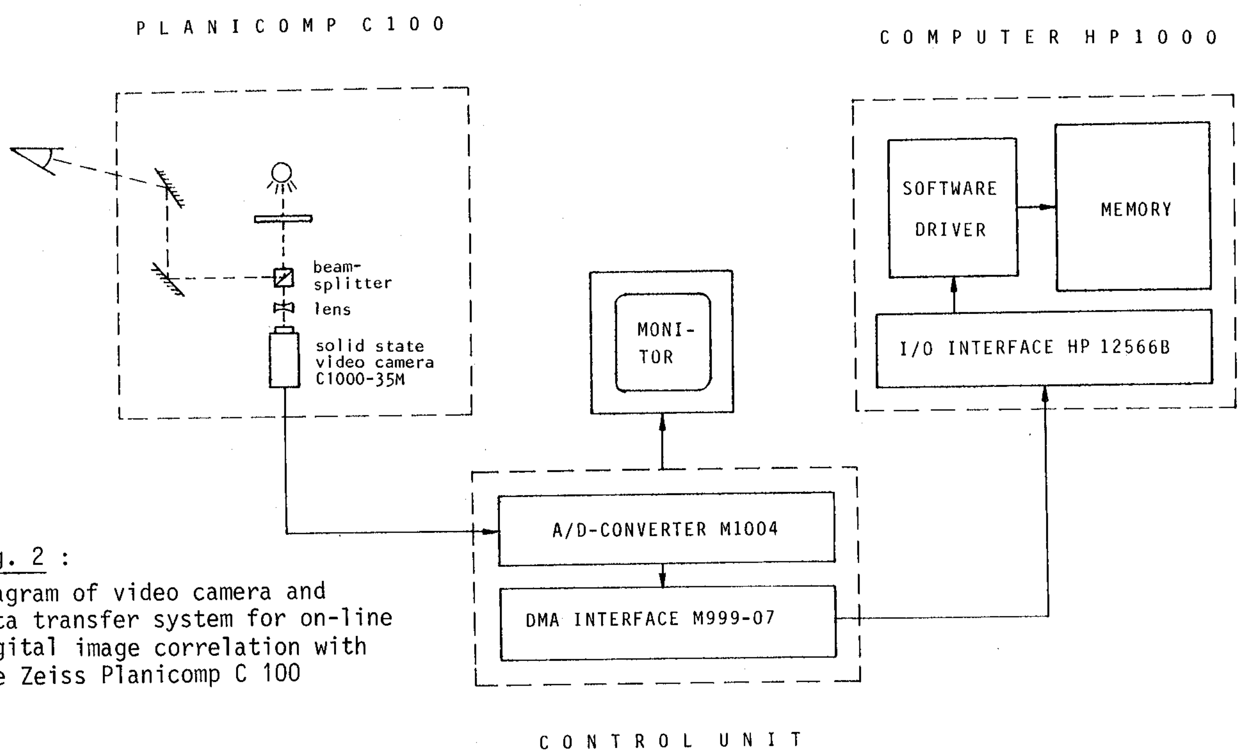

Fig. 2 :
Diagram of video camera and data transfer system for on-line digital image correlation with the Zeiss Planicomp C 100

There the image correlation takes place with the previously described program which has been reprogrammed for this particular computer. At present the total computation takes, as explained before, about 7 - 10 sec, for 3 - 4 iterations, with windows of 32 x 32 pixels. This is still somewhat slow; it is sufficient, however, to continue the investigations.

The system has just been implemented and goes presently through the first tests and calibration procedures.

It has been pointed out before that the operator is practically not disturbed at all by the instantaneous digitising of the central parts of his field of view. In addition, there are a few most welcome operational aspects. The operator will put the measuring mark approximately on the area to be digitised. This implies that a good initial approximation is obtained. It also implies that the operator controls whether the area is suited for correlation. In difficult cases he would

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take over. It is also noticeable that the results of image correlation directly refer to the image coordinate systems. These conditions correspond very well with the philosophy that the operator is more and more relieved from the task of precise measurements, but that he is fully controlling and monitoring the operations. The system provides also a very good basis for expanding its tasks (see next section).

5. Application and further development

5.1 Immediate application

The described system of digital image correlation is ready for on-line operations with the Planicomp C 100. Although elaborate testing and calibration still has to be done, we intend to investigate immediately the practical application. At present, the system is restricted to deliver parallaxes only. This performance alone opens already a wide range of application. There are 4 areas of direct application which will be investigated in the near future.

a) Automatic measurement of y-parallaxes for relative orientation.

b) Automatic measurement of x-parallaxes, and their conversion to heights, for digital elevation models.

c) A particularly important application will be deformation measurements. The method allows fast measurement of a dense network of arbitrary points in a pair (or several pairs) of photographs. The automatic measurements are processed through conventional analytical procedures to ground coordinates. If then the points, which are not signalized, are transferred by digital image correlation to photographs taken at a later epoch the original points are identified and can again be correlated in the new pair(s) of photographs. Thus displacements or deformation can be assessed.
The main point is, that (relative) precision of the method is extremely high (1 µm), and that no signalisation of points is required. (The absolute accuracy still depends on the well known image errors and will only reach perhaps 5 or 6 µm.) The only precondition is that vegetation cover will not upset the precision. Application for tectonic deformations and for assessing subsidence in open or underground mining areas are feasible, apart from close range application.

d) Point transfer for aerial triangulation. This application has been the primary object of the development. It is of great importance as it offers an elegant solution of the still cumbersome task of point transfer. The method not only is capable of superior accuracy, it also does away with multiple measurements of the same photograph in combination with all adjacent overlapping photographs. Instead, only the digitised (or processed) image windows are stored and recalled for multiple use. The transferred points are identified by image coordinates, therefore they fit smoothly into any analytical system of aerial triangulation.

5.2 Further development

Our method of image correlation is, at present, restricted to sensing parallax and to transferring points. This is, however, only the beginning of further developments which offer themselves immediately and which are potentially of great practical importance, even when restricting the considerations to the classical tasks of aerial photography.

The first extension refers to an analysis of the residual gray-values after transformation of one window onto the other. The purpose is to analyse whether higher degree transformations should be applied, in order to improve the result. This may become important with the extension of the method, as described below, when larger windows will possibly be used.

The main extension will refer to the identification and location of detail within a window, by image correlation. The simplest case will be the automatic identification and measurement of signalized points. Beyond that the identification of various detail features is feasible. [1] The main point is that, at least initially, the problem is not approached by pattern recognition techniques. In connection with an analytical plotter the operator will classify the type of target in question.

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[1] *See also Mikhail, Photogrammetric Target Location to Sub-Pixel Accuracy in Digital Images, page 217*

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The last step of extension, as far as the concept goes for the time being, will be line following by shifting the windows along distinguishable line features.

We intend to pursue such developments. They seem to be within immediate reach. In connection with analytical plotters it is expected to come to operation procedures rather soon, provided it will be possible to develop fast enough computing procedures which will allow real time operations. We are convinced, however, that a development has been opened which will have great practical implications concerning the conventional tasks of photogrammetric measurement and mapping. The method will fit into the conventional process of photogrammetric restitution and will not require very costly nor extra equipment.

6. Conclusion

The described image correlation procedure for local image areas is only the beginning of a, hopefully, far reaching development. Nevertheless, two prime results emerged already now: It has been shown that image correlation is capable of unexpectedly high accuracy. And it does not require too costly equipment. In connection with analytical plotters it could go into operational application very soon, and it would fit into the conventional modes of operation. The further development is expected to have a great impact on the practice of obtaining digital and graphical photogrammetric products. Its high accuracy performance and its operational characteristics might be able to transgress the conventional range of applications of photogrammetry.

The described development has been entirely independent. However, it should be mentioned that there are other developments of a similar nature elsewhere. In particular reference is made to developments at the Defence Mapping Agency, at Purdue University and other remote sensing centres in the United States. Relevant developments are also known from Great Britain, referring to automatic identification of ground control points (ground features) in Landsat multispectral images. In the field of remote sensing digital image correlation is most widely and generally applied. It seems, however, that the special requirements of photogrammetry relating to high precision and to geometry have not been particularly pursued to the extent that they could be taken over directly.

Abstract

The paper presents a development, the aim of which is the high precision digital correlation of small image areas.

The method attempts the matching of the matrices of gray values of the related image windows by transformation of higher degree which is to remove the geometrical distortions of the imaging process, relief displacements included, as well as systematic radiometric differences. The special approach determines the unknown transformation parameters directly by a least squares solution by minimizing the remaining differences of gray values. The solution is obtained by iteration, because of non-linearity. The correlation coefficient serves as quality parameter.

Experimental and theoretical results have shown that the displacement parameters of the transformation, i.e. the parallax determination, can reach a precision of 1 µm or better in the image (1/50 to 1/100 pixel size), depending mainly on the signal to noise ratio and on window size.

Finally the installation of CCD Arrays in the Planicomp C 100 is described, and an outlook is given on operational application of digital image correlation, up to automatic measurement of signalized points.

HOCHGENAUE DIGITALE BILDKORRELATION

Zusammenfassung

In dem Vortrag wird eine eigene Entwicklung vorgestellt, welche die möglichst genaue digitale Bildkorrelation kleiner Bildausschnitte zum Ziel hat.

Die Methode beruht darauf, eine möglichst genaue Übereinstimmung der Grauwertmatrizen der zu korrelierenden Bildausschnitte durch Transformation höheren Grades zu erreichen, welche die geometrischen Abbildungsverzerrungen einschließlich der Reliefversetzungen sowie systematische radiometrische Unterschiede beseitigt. Der spezielle Ansatz bestimmt die Transformationsparameter direkt als

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Unbekannte in einer Kleinste-Quadrate-Lösung durch Minimierung der verbleibenden Grauwertdifferenzen, wobei die Lösung wegen Nichtlinearität iterativ erfolgen muß. Der Korrelationskoeffizient dient dabei nur als Gütemerkmal.

Es wird experimentell und theoretisch gezeigt, daß dabei die Verschiebungsparameter der Transformation, d.h. die Parallaxenbestimmung, in Abhängigkeit hauptsächlich vom Signal-Rausch-Verhältnis und von der Fenstergröße, Genauigkeiten von bis zu 1 µm oder besser im Bild (1/50 bis 1/100 der Pixelgröße) erreichen.

Schließlich wird die Ausrüstung des Planicomp C 100 mit CCD Arrays beschrieben und ein Ausblick auf operationelle Anwendungen der digitalen Bildkorrelation in der Photogrammetrie bis zur automatischen Messung signalisierter Punkte gegeben.

CORRELATION NUMERIQUE D'IMAGES DE HAUTE PRECISION

Résumé

L'exposé présente une méthode mise au point dans les services de l'auteur et dont le but est la corrélation numérique la plus précise possible entre des fragments d'image de petites dimensions.

Cette méthode consiste à apparier le plus précisément possible les matrices des niveaux de gris produites à partir des fragments d'image à mettre en corrélation, cet appariement se faisant par transformation au plus haut degré afin d'eliminer les déformations géométriques des images, les décalages du relief et les différences radiométriques systématiques. L'application détermine directement les paramètres de transformation en tant qu'inconnues par la méthode des moindres carrés et par minimisation des différences de gris résiduelles, le calcul devant s'effectuer par itération en raison de la non-linéarité. Le coefficient de corrélation sert uniquement de symbole de qualité.

Les résultats expérimentaux et théoriques ont montré que les paramètres de translation de la transformation, c'est-à-dire la détermination des parallaxes, atteignent des précisions de 1 µm ou meilleures (1/50 à 1/100 de la taille du pixel) en fonction principalement du rapport entre le signal et le fond et de la taille de la fenêtre.

L'exposé se termine sur une description de l'équipment Planicomp C 100 avec circuits CCD Arrays et sur une vue d'ensemble des applications opérationnelles de la corrélation numérique d'images en photogrammétrie jusqu'à la mesure automatique de points signalisés.

CORRELACION DIGITAL DE IMAGENES, DE SUMA PRECISION

Resumen

En la conferencia se presenta un método, desarrollado en el instituto del autor, cuya finalidad es la correlación digital lo más precisa posible entre áreas de la imagen de pequeñas dimensiones.
El método tiene por objeto lograr la coincidencia lo mas precisa de las matrices del nivel de grises correspondientes a las áreas en la imagen que se deseen correlacionar y ello mediante una transformación de grado superior, la cual elimina las distorsiones geométricas de las imágenes, los desplazamientos de relieve a la vez que las descrepancias radiométricas sistemáticas. El método matemático especial determina los parámetros de transformación directamente como incógnitas en una solución de mínimo cuadrados por reducción al mínimo de las diferencias residuales de los niveles de grises, siendo necesario conseguir la solución por vía iterativa, debido a la no-linealidad. El coeficiente de correlación no sirve sino de índice de calidad.

Se demuestra tanto experimental como teoreticamente que los parámetros de translaciónes de la transformación, o sea la determinación de las paralajes, alcanzan en la imagen exactitudes de 1 µm y superiores (1/50 hasta 1/100 de pixel o sea del área explorada), en función principalmente de la relación señal/ruido y del tamaño de la ventanilla.
Además y como conclusión se describe el Planicomp C 100 provisto de detectores CCD y se estudian las perspectivas que brindarán las aplicaciones operacionales de la correlación digital de imágenes en fotogrametría hasta la medición automática de puntos señalizados.

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