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THE RELIABILITY OF BLOCK TRIANGULATION

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1. Introduction

1.1 During the last 10 years aerial triangulation has become a powerful tool for point determination. Main reason is the rigorous application of adjustment theory, which enabled a simultaneous orientation of images and thus increased the accuracy by one order of magnitude. The refinement of the mathematical model based on this development, with the aim to compensate systematic errors, lead to a further increase of the accuracy by a factor 2 - 3. Today one can reach a precision of the adjusted coordinates of 2 - 3 µm, measured in photoscale, if the full potential is used to advantage. This result is confirmed by various controlled tests.

1.2 It is at the same time pleasing and amazing that these accuracies are also achieved in normal application, as the theoretical studies on the precision of block triangulation were based on very simplified assumptions about the stochastical properties of the image or model coordinates. It is true that the discrepancies between theoretical predictions and empirical results have pushed forward the development of methods for compensation of systematic errors and have lead to a deeper insight into the powerful tool of self calibration. But the effects of unmodelled errors, especially gross errors, on the adjustment of photogrammetric blocks have not been studied thoroughly until a few years ago.

1.3 However, each block adjustment has to cope with a certain percentage of gross errors, which in general are found and eliminated by an analysis of the residuals. But, as known to everybody who has once cleaned a block, there does not exist an objective and commonly accepted criterium about when to stop with the elimination of possibly erroneous observations.

Thus there may stay undetected gross errors which are hoped not to deteriorate the result too much. This immediately leads to the reliability of adjusted coordinates being the intrinsic problem of point determination.

1.4 Two tasks have to be solved:

- One needs methods for the detection and elimination of gross errors. Automatic procedures have to take into consideration the different types of gross errors, thus have to be able also to handle large gross errors. They therefore cannot be reduced to the application of a statistical test. The development of efficient strategies seems to converge to a three-step procedure treating large, medium-sized and small gross errors by pre-error detection, automatic weighting and statistical procedures resp. (cf. the papers presented at the Commission III Symposium of ISP, 1982, in Helsinki).
- The detectability of gross errors and the influence of non-detectable gross errors, i.e. the reliability according to Baarda, onto the result of the block adjustment has to be investigated with respect to the project planning. Here the type of test for detecting very small gross errors will have a large influence and will require a statistical description of reliability.

This paper is supposed to motivate and describe the concept of reliability. Based on the result of comprehensive studies recommendations for the project planning are given, which complete the known methods for improving the precision of photogrammetric coordinates.

2. The concept of reliability

2.1 The theory of reliability is part of a concept for evaluating the quality of adjustment results, which was developed by W. Baarda for the use in geodetic networks. The notion quality according to Baarda includes precision and reliability. Fig. 1 shows the interrelations between the different parts of the theory.

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The evaluation of the precision consists in the comparison of the covariance matrix Q_{kk} of the adjusted coordinates with a given matrix H_{kk}, also called criterion matrix. It is required that the error ellipsoid described by Q_{kk} lies inside the error ellipsoid described by H_{kk} and that is as similar to H_{kk} as possible. Thus it is checked whether a required measure of precision is reached.

We are only concerned with the reliability here. Baarda distinguishes internal and external reliability. Internal reliability is the controllability of the observations, described by lower bounds for gross errors, which can just be detected with a given probability. The effect of non-detectable gross errors onto the result is described by factors for the standard errors of the coordinates, which indicate up to which measure the coordinates may be deteriorated in the worst case. These factors describe the external reliability or the sensitivity of the unknown coordinates. Obviously there is a close connection to the evaluation of the precision.

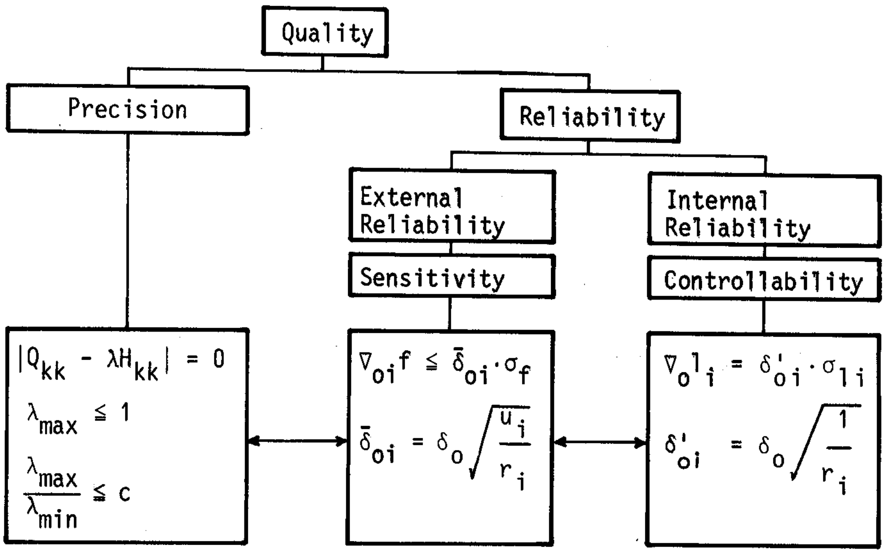


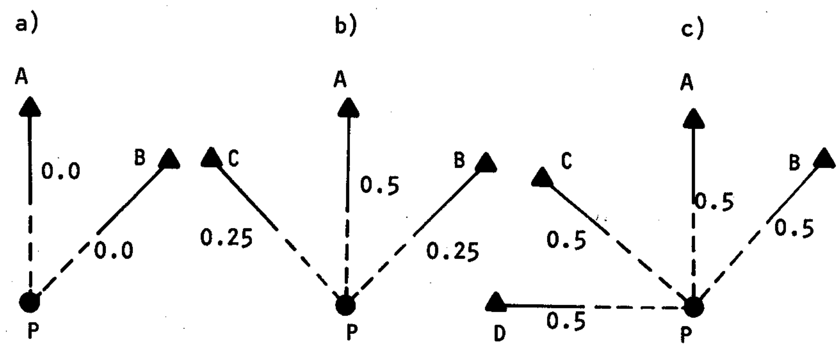
Fig. 1: Evaluation of quality according to W. Baarda (1967, 1968, 1971, 1976)

Remark: Baarda himself used the notion "accuracy" for describing precision and reliability. As in photogrammetric application the notion accuracy is too much associated with the specification of estimated standard deviations, quality seems to be a more proper term for also describing non-detectable errors in the mathematical model and their effects.

2.2 Before a mathematical definition of the reliability, three examples show that the conditions for a good reliability are closely related to the local geometry. Fig. 2 shows network diagrams of forward intersections, which differ by one observation each. They are to demonstrate the influence of additional observations

Fig. 2:

Forward intersections with different redundancy, good geometry, redundancy numbers, in all directions of c) errors locatable



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on the strength of the geometry. In Fig. 2a) the observations are not controllable, i.e. they are necessary for the determination of the coordinates. In Fig. 2b) each observation is controllable, i.e. it is not necessary for the determination of the coordinates. On the other hand, errors in the observation are not locatable or identifiable, i.e. each is necessary for the control of the others. In Fig. 2c) finally, errors in the observations are locatable, i.e. each is not necessary for the control of the others. Any further observation would be superfluous for the detection of a single gross error. The increase of the number of rays goes parallel with an increase of the strength of the geometry, which may be described by the sequence of the common terms: determination-control-location or identification.

Obviously the network design in Fig. 2c) is chosen appropriately to reach a reliable determination of the coordinates. This is different in figures 3 and 4.

Fig. 3:
Forward intersections with different redundancy, poor geometry, redundancy numbers, errors in directions C and D in c) not locatable

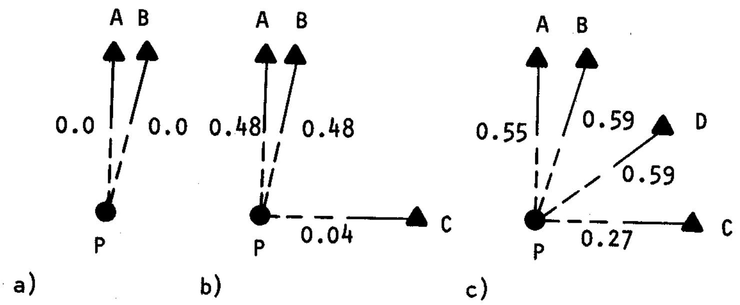
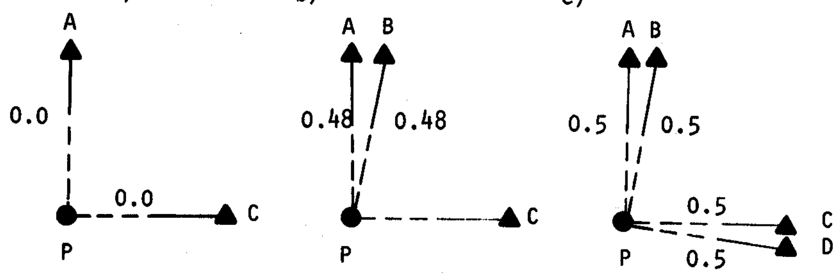


Fig. 4:
cf. Fig. 3, but errors in directions A, B, C and D in c) not locatable



In Fig. 3a) the determination of point P is weak, therefore the direction C in Fig. 3b) is not controllable. The directions A and B control each other. As a consequence gross errors in directions C and D in Fig. 3c) and in all directions of Fig. 4 practically cannot be located. Any additional direction in Fig. 4 cutting the existing directions with an angle of about 45° enables a location of a gross error in any of the directions.

As these configurations may occur also in networks with high redundancy, it is obvious that the local geometry is decisive for the reliability of the point determination. Such situations also arise in bundle blocks with 20 % sidelap where the points in the middle of the strips are measured only in 3 images. The x-coordinates (x parallel to direction of flight) are controllable here, but possibly wrong beams cannot be identified (cf. geometry of Fig. 2b)).

2.3 This discussion of the geometric properties of a network design can be rendered precise using the information adjustment theory and mathematical statistics offer.

Notation: Scalars and vectors are written in small letters, matrices in capital letters, A' is the transpose of A, stochastical values are underscored, $\hat{\underline{x}}$ and $\widehat{\underline{\nabla l}}$ are estimates for x and ∇l respectively.

Let the block adjustment be given by the linearized error equations

$$\underline{l} + \underline{v} = A\,\hat{\underline{x}} + a_0 \qquad P_{ll} \qquad (1)$$

with the vector $\underline{l} = (\underline{l}_i)$ containing the observations $\underline{l}_i$, the corresponding vector $\underline{v} = (\underline{v}_i)$ of the residuals, the error equation or design matrix A with the error equation vectors a_i', the estimated vector $\hat{\underline{x}}$ of the unknown parameters (coordinates, transformation parameters, possibly additional parameters) and a constant vector a_0, resulting from the linearization. The weight matrix P_{ll}, being the inverse of the weight-coefficient matrix Q_{ll}, is supposed to be known.

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With the solution $\hat{\underline{x}} = (A' P_{ll} A)^{-1} A' P_{ll} (a_o - \underline{l})$ and the weight coefficient matrix

$$Q_{vv} = Q_{ll} - A (A' P_{ll} A)^{-1} A' \qquad (2)$$

of the residuals $\underline{v}$, which are used for error detection, they are directly related to the observations by

$$\underline{v} = - Q_{vv} P_{ll} (\underline{l} - a_o). \qquad (3)$$

The matrix $Q_{vv} P_{ll}$ is idempotent, i.e. $(Q_{vv} P_{ll})^2 = Q_{vv} P_{ll}$. Using the eigenvalue decomposition of $Q_{vv} P$ it can be shown that the rank and the trace of $Q_{vv} P_{ll}$ are equal

$$rk (Q_{vv} P_{ll}) = tr (Q_{vv} P_{ll}) = \sum_{i=1}^{n} (Q_{vv} P_{ll})_{ii} = r \qquad (4)$$

to the redundancy $r = n - u$ of the system. The diagonal elements $(Q_{vv} P)_{ii}$ obviously show the distribution of the redundancy onto the observations.

The <u>redundancy number</u>

$$\boxed{r_i \overset{def}{=} (Q_{vv} P_{ll})_{ii}} \qquad (5)$$

is the contribution of $\underline{l}_i$ to the total redundancy r.

The redundancy numbers range from 0 to 1. Observations with $r_i = 1$ are fully controllable, whereas observations with $r_i = 0$ cannot be checked at all. The average value, the relative redundancy $\bar{r} = r/n$, for photogrammetric blocks is about 0.2 to 0.5. An average value of 0.5 already indicates a rather stable block. Single redundancy numbers, however, easily reach values below 0.1, indicating a very weak local geometry.

Using the r_i, we are now able to calculate the influence ∇v_i of a single gross error ∇l_i in the observation $\underline{l}_i$ on the corresponding residual $\underline{v}_i$

$$\nabla v_i = - r_i \nabla l_i \qquad (6)$$

(∇ designates "gross error" or, more general, a deviation from the assumed mathematical model).

Eq. (6) shows that only a small part (about 50 % down below 10 %) of the original gross error ∇l_i is revealed in the residual v_i. However, only in case the diagonal element $(Q_{vv} P)_{ii} = r_i$ of $Q_{vv} P_{ll}$ (cf. eq. (2)) is larger than all other elements of the i-th column, a gross error in $\underline{l}_i$ will influence $\underline{v}_i$ more than the other residuals, i.e. only in that case ∇l_i can be expected to be locatable using the largest residual as indicator.
Eq. (6), furthermore, can be used in practical error detection procedures. With the knowledge of the local geometry, i.e. with r_i, one can obtain an estimate $\widehat{\nabla l}_i$ for the size of the original gross error ∇l_i

$$\widehat{\nabla l}_i = - \underline{v}_i / r_i . \qquad (7)$$

This simplifies the evaluation of the residuals. The spatial distribution of the redundancy onto the block can be described using the redundancy numbers and gives a first insight into the controllability of the observations. Fig. 2 - 4 show the values r_i, which demonstrate that the visual evaluation of the design quality is confirmed by the numerical values.

<u>Remark:</u> The redundancy numbers r_i do not give any information about the ability to identify or locate gross errors. However, in general gross errors in an observation are locatable, if $\underline{l}_i$ is not necessary for the control of the other observations. In this case all correlation coefficients ϱ_{ij} of $\underline{v}_i$ and $\underline{v}_j$ $(j \neq i)$ are $\neq \pm 1$.

2.4 <u>Testing</u> the observations based on the residuals has to consider the different precision of the residuals. With the standard deviation

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$$\sigma_{vi} = \sigma_0 \sqrt{Q_{v_iv_i}} \overset{1)}{=} \sigma_{li} \sqrt{r_i} \qquad (^{1)}\text{provided } P = \text{diag}(p_i)) \qquad (8)$$

of the i-th residual $\underline{v}_i$ we obtain the standardized residual

$$\underline{w}_i = \frac{-\underline{v}_i}{\sigma_{vi}} = \frac{\widehat{\nabla l}_i}{\sigma_{\widehat{\nabla l}_i}} = \frac{-\underline{v}_i \sqrt{p_i}}{\sigma_0 \sqrt{r_i}} = \frac{\widehat{\nabla l}_i \sqrt{r_i}}{\sigma_{li}} \sim N(0,1) \qquad (9)$$

which is used as test statistic. It is at the same time a test of the estimated size $\widehat{\nabla l}_i$ of the gross error (cf. eq. (7)). Here σ_0 is assumed to be known. Moreover, if it is assumed that the observations are normally distributed, then the test statistic $\underline{w}_i$ also is normally distributed with expectation 0 and variance 1.

Remark: If there is no a priori information about the precision of the observations, thus if σ_0 is not known, instead of $\underline{w}_i$ the test statistic

$$\bar{\underline{w}}_i = \frac{-\underline{v}_i}{\hat{\underline{\sigma}}_{oi} \sqrt{Q_{v_iv_i}}} = \frac{-\underline{v}_i \sqrt{p_i}}{\hat{\underline{\sigma}}_{oi} \sqrt{r_i}} \sim t(r-1) \qquad (10)$$

can be used, which follows a student distribution with r-1 degrees of freedom. The variance

$$\hat{\underline{\sigma}}^2_{oi} = \frac{[vvp] - \underline{v}_i^2\, p_i / r_i}{r-1} \qquad (11)$$

is an estimate for σ_0^2 and identical to $\hat{\underline{\sigma}}_0^2$ in an adjustment without observation l_i (cf. Förstner, 1980, eq. (12)). This test statistic $\bar{\underline{w}}_i$ is functional dependent on the one given by Pope (1975), which is τ-distributed (cf. Grün, 1982, eq. (22b)). $\bar{\underline{w}}_i$, however, is suited as well as $\underline{w}_i$ for the following derivations, as in both cases the non-central distribution is known.

The test of the observations, the "data-snooping" proposed by Baarda, consists in comparing the absolute value $|\underline{w}_i|$ of the test statistic with the critical value k, which depends on the preset significance level $S = 1 - \alpha_0$. If the standardized residual exceeds the critical value k, the corresponding observation l_i is suspected to be erroneous. In a practical procedure one of course will only check the observations with the largest test statistics and take into consideration the interrelation between the residuals, as a single large gross error will lead to many test statistics which exceed the critical value. As can be seen from eq. (9), a large $\underline{w}_i$ also can be caused by a wrong weight. $\underline{w}_i$ must be accepted, if only being a little larger than the critical value k; since the normal distribution - at least in principal - allows deviations of any size from the mean value. The probability that the test statistic exceeds the critical value, if the observations are not erroneous, and therefore leads to an erroneous decision of I. type, is the significance number α_0, which usually is chosen small (e.g. 5 %, 1 % or 0.1 %).

2.5 On the other hand, gross errors may stay undetected. The power β_i of the test, i.e. the probability of detecting a gross error, and the probability for this erroneous decision, an error of II. type, depend on the size ∇l_i. The gross error ∇l_i changes the test statistic $\underline{w}_i$ by $\delta_i = \nabla w_i$, i.e. ∇l_i shifts the probability density function of $\underline{w}_i$ by δ_i, thus leads to a non-central distribution (cf. Fig. 5) with non-centrality parameter δ_i.

As the size of gross errors is unknown, Baarda proposed to require a minimum probability β_0 to detect a gross error, i.e. to start with a minimum power β_0 of the test and to determine the lower bound $\nabla_0 l_i$ for a gross error in the observation l_i which can be detected with a probability $\beta > \beta_0$. As can be seen from Fig. 5, a given lower bound β_0 leads to a lower bound $\delta_0 = \nabla_0 w_i$ for the non-centrality parameter. We will use $\delta_0 = 4$, for simplicity, which corresponds to a significance level $1 - \alpha_0 = 99$ % and a probability $\beta_0 = 93$ % for error detection. Table 1 shows the dependency of β_0 on the critical value k for a given $\delta_0 = 4$. In accordance with experience, gross errors can be found more easily, the smaller the critical value k is chosen.

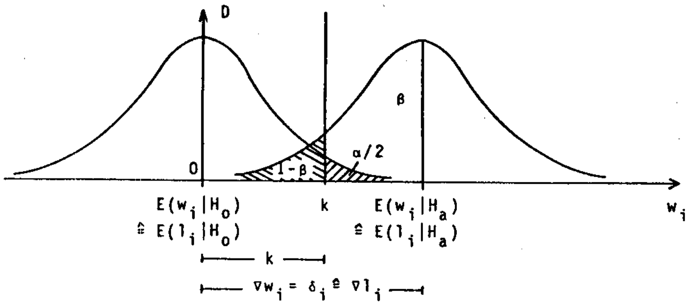

Fig. 5: Probabilities of wrong decisions of I. and II. type when using data-snooping

$S = 1 - \alpha_o$	k	β
99.9 %	3.29	76 %
99.7 %	3.00	84 %
99.0 %	2.56	93 %
95.0 %	1.96	98 %

Table 1:

Significance level $S = 1 - \alpha_o$, critical value k and power β_o of test for given non-centrality parameter $\delta_o = 4$.

From eq. (9) now the lower bound $\nabla_o l_i$ for gross errors, which are detectable with a probability $>\beta_o$ can be derived:

$$\nabla_o l_i = \delta'_{oi} \cdot \sigma_{li} \; ; \; \delta'_{oi} = \delta_o \sqrt{\frac{1}{r_i}} \tag{12}$$

The value δ'_{oi} is the factor for σ_{li} giving a minimum size $\nabla_o l_i$ of a just detectable gross error in l_i. The lower bounds $\nabla_o l_i$ or the factors δ'_{oi} designate the controllability of the observations or the internal reliability according to Baarda. They essentially depend on the redundancy numbers r_i, i.e. on the local geometry. Redundancy numbers between 0.1 and 0.5 lead to lower bounds for detectable gross errors between $\nabla_o l_i = 6\ \sigma_{li}$ and $\nabla_o l_i = 13\ \sigma_{li}$ or to controllability factors between $\delta'_{oi} = 6$ and $\delta'_{oi} = 13$. Thus gross errors much larger than the 3-fold standard deviation may stay undetected and may falsify the result.

Remark: A similar line of thought leads to the notion of locatability or identifiability, describing the ability to correctly locate or identify gross errors. Requirements for a high locatability analogously lead to requirements for the correlation coefficients of the residuals. This aspect will not be treated here.

2.6 All not detected gross errors contaminate the result of the adjustment. The influence $\nabla_{oi} x_j$ of a gross error of size $\nabla_o l_i$ on the unknown $\hat{x}_j$ can directly be obtained using $\hat{x} = (A'P_{ll}A)^{-1}A'P_{ll}(a_o - \underline{l})$:

$$\nabla_{oi} x_j = -((A'P_{ll}A)^{-1}A'P_{ll})_{ji}\ \nabla_o l_i \tag{13}$$

The values $\nabla_{oi} x_j$ give conspicuous insight into the sensitivity of the result and may be useful in small systems.

There are, however, several reasons to use a different measure:

- The calculation of all n x u values $\nabla_{oi} x_j$ is prohibitive in large blocks.
- In most cases the influence of non-detectable gross errors on the orientation parameters or even on additional parameters is of no interest.
- In free blocks, without any control points, the influence values $\nabla_{oi} x_j$ depend on the coordinate system.

Therefore Baarda proposed to use the standardized length $\bar{\delta}_{oi}$ of the vector $\nabla_{oi} k$, a subvector of $\nabla_{oi} x$, containing the influence on the coordinates $\hat{k}$ of the new points

$$\bar{\delta}_{oi} = \| \nabla_{oi} k \| = \sqrt{\nabla_{oi} k'\ Q_{kk}^{-1}\ \nabla_{oi} k / \sigma_o}. \tag{14}$$

It is a measure for the total deformation of the block, caused by a gross error $\nabla_o l_i$ in observation $\underline{l}_i$. This seemingly abstract measure for the deformation at the same time gives an upper bound $\nabla_{oi} f$ for the influence of gross errors

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$\nabla l_i < \nabla_o l_i$ on an arbitrary function $\underline{f} = f(\hat{\underline{k}})$ of the coordinates $\hat{\underline{k}}$. Using Cauchy-Schwarz's identity, it can be shown that with the standard deviation σ_f of $\underline{f}$ the influence is bounded:

$$\boxed{\nabla_{oi} f \leq \bar{\delta}_{oi} \cdot \sigma_f} \quad (15)$$

For the special functions $\underline{f} = \underline{x}_j$, $\underline{f} = \underline{y}_j$, $\underline{f} = \underline{z}_j$ one obtains

$$\boxed{\nabla_{oi} x_j \leq \bar{\delta}_{oi} \cdot \sigma_{xj} \;,\; \nabla_{oi} y_j \leq \bar{\delta}_{oi} \cdot \sigma_{yj} \;,\; \nabla_{oi} z_j \leq \bar{\delta}_{oi} \cdot \sigma_{zj}} \quad (16)$$

resp. Thus the coordinates $\underline{x}$, y and z are not contaminated more than by $\bar{\delta}_{oi}$ times their standard deviation. The factors $\bar{\delta}_{oi}$ describe the <u>sensitivity</u> of the result of the <u>external reliability</u> according to Baarda. A practical formula for calculating the values δ_{oi} is given by Klein/Förstner in Seminar (1981).

3. <u>The theoretical reliability of photogrammetric blocks</u>

3.1 On the basis of the reliability theory described in the previous section, several photogrammetric blocks were investigated at the Institute of Photogrammetry, Stuttgart, in order to obtain information about the dependency of the internal and external reliability on different project parameters. These were especially:

- the control point distribution
- the degree of overlap
- the density and the distribution of the tie points and
- the size of the blocks.

The investigation used simulated regular square shaped blocks with bundles and independent models, single blocks (designated with S) with 20 % sidelap and double blocks (designated with D) with either 60 % sidelap or consisting of two single blocks flown crosswise. The blocks with independent models use 4 or 6 single or twin points per model. The bundle blocks have 9 single or twin points and 25 single points per image. The number of points per unit is added to the S or D to describe the block concerned. The horizontal control points are situated at the perimeter of the blocks (cf. Fig. 6). The vertical control varies for

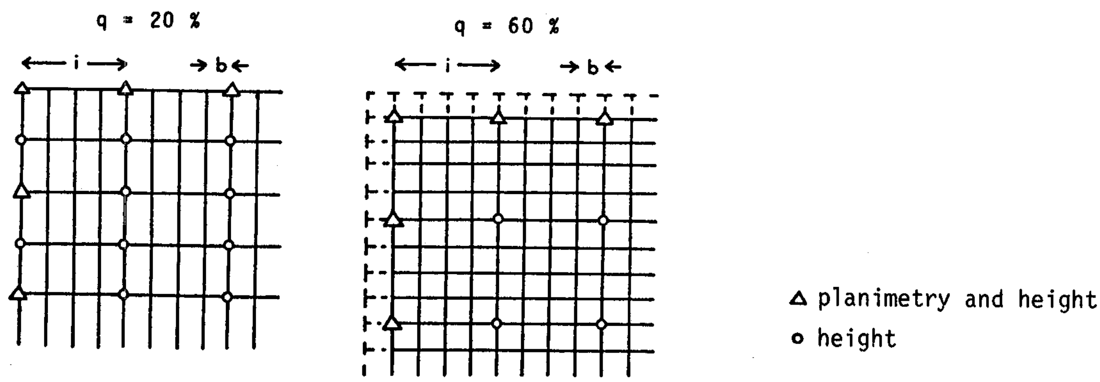

Fig. 6: Control point distribution

single and double blocks. The height of single blocks is stabilized by chains, the vertical control points in double blocks form a regular grid. The control point interval varies from 2 to 20 baselengths b. The precision of all observations, including the coordinates of the control points, is assumed to be equal, with one exception: the x- and y-coordinates of the projection centres in independent model blocks are assumed to have double the standard deviation.

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Before we investigate the influence of the different block parameters on the reliability, we have a look at two examples. Fig. 7 - 9 show two representative blocks with independent models and with bundles respectively. For symmetry reasons only 1/4 of the blocks is plotted. The redundancy numbers, r_i, the controllability factors δ'_{oi} and the sensitivity factors $\bar{\delta}_{oi}$ are given one upon another. The values are given separately for the planimetry and height of independent model blocks and for the x- and y-coordinates of bundle blocks.

3.2 The model blocks S 8 (cf. Figs. 7 and 8) have 4 twin points in the corner of each model. The reliability figures are identical for the two points of a group and therefore only given once. The figures suggest to consider the interior parts, the border parts and the control points separately.

The redundancy numbers r_i in the interior of the blocks are about 0.5. This proves the block to be very stable. Gross errors larger than $\delta'_{oi} \cdot \sigma_{li} = 5.6\ \sigma_{li}$ can be detected with the data snooping. Undetectable gross errors, however, falsify the coordinates of the new points only up to 3 times their standard deviation ($\bar{\delta}_{oi} \leq 3$). The reliability is fully acceptable.

This is different at the border parts of the blocks, especially at the borders with the short model sides. They are determined not very reliable, with sensitivity factors $\bar{\delta}_{oi}$ around 5. The influence of the control points on the reliability is negligible.

The coordinates of the control points, which are introduced as observations, are worst controllable. Even with moderate control point distance i = 6b only 7 or 12 % of gross errors show up in the residuals (r_i = 0.07 or 0.12 resp.). Gross errors must be larger than 6 or 15 (!) times the standard deviation σ_{xy} or σ_z of the model coordinates to be detectable.

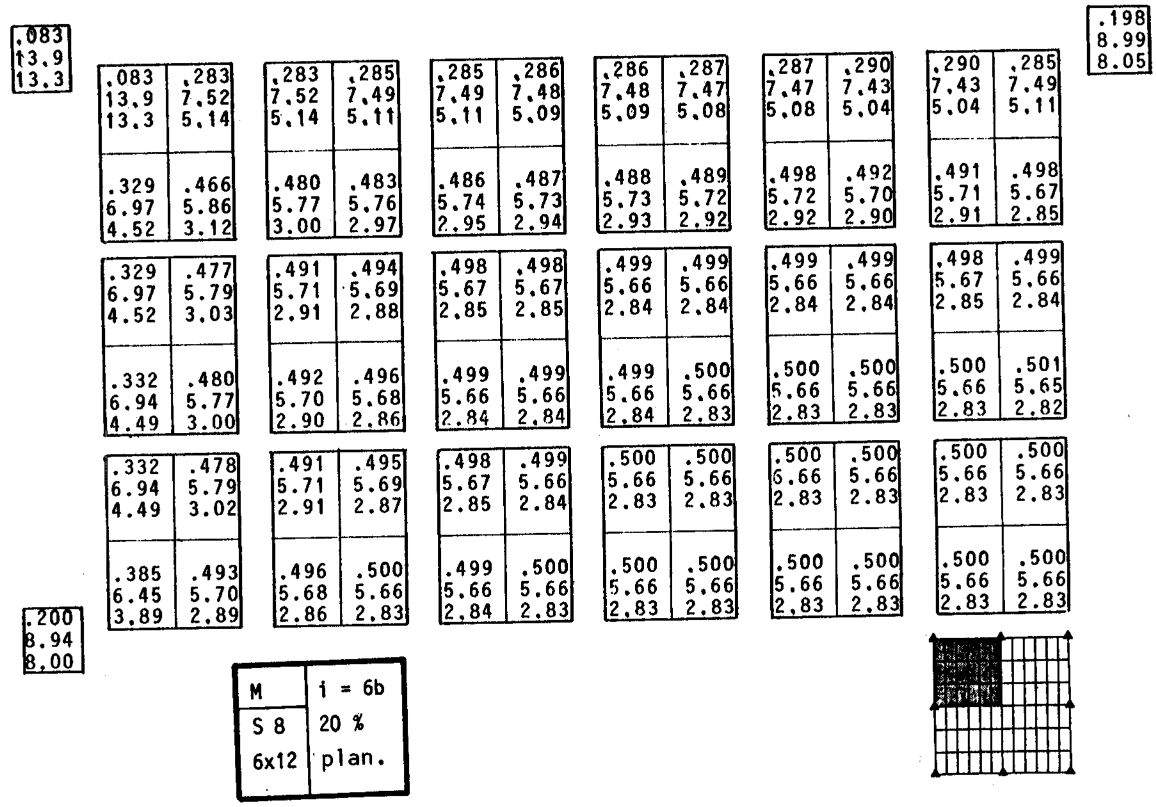
3.3 Bundle blocks reveal a similar structure of the reliability. In the images of block S 18 shown in Fig. 9 double points are measured at the 9 standard positions. Also here the values suggest to consider the interior and the border parts separately. This statement is not influenced by the points in the middle of the strips, which occur in all images. At these points only 1/6 of gross errors in the x-coordinates reveal in the residuals (r_i = 1/6), which moreover cannot be located (cf. section 2.2). At the border parts some observations are not controllable at all (δ'_{oi} and $\bar{\delta}_{oi} = \infty$). These points would be single points in an adjustment with independent models and in a previous analytical relative orientation would be controllable only in y-directions. The points in the overlap zone of the adjacent strips, however, are well determined with sensitivity factors $\bar{\delta}_{oi}$ below 3.

The control points are as weakly controllable as in model blocks, discussed above. Here also the height control points are less controllable than the horizontal control points with factors $\bar{\delta}_{oi}$ of 14.6 or 11. versus 13. or 8. at the corners or the borders respectively.

A direct comparison of the reliability of bundle and independent model blocks is not possible, as the controllability and sensitivity factors refer to different types of observations and as the structure of the precision of the new points is different.

Blocks with single tie points are worse reliable (not shown). Here the controllability factors are larger for the interior and the border parts by a factor 1.2 - 1.5 and 2 respectively. In blocks with independent models with only 4 tie points the lower bounds for detectable gross errors reach values of 22 σ_l at the borders of the blocks. The sensitivity factors $\bar{\delta}_{oi}$ of blocks with single tie points are larger by about a factor 1.1 - 1.5 and 1.5 - 3.0 for the interior and the border parts respectively. The measurement of double tie points thus leads to a rather high reliability. Moreover, in case a tie point has to be eliminated the connection is not lost and still can be controlled.

3.4 The two examples give a first impression about the reliability of photogrammetric coordinates, but provide no information about the dependency on the block parameters. Main result, however, is the high homogeneity of the values in the interior of the blocks. This indicates that the values are independent of the block size and also of the shape of the block. It can be expected that the reliability will be affected by an increase of the tie point density or the over-



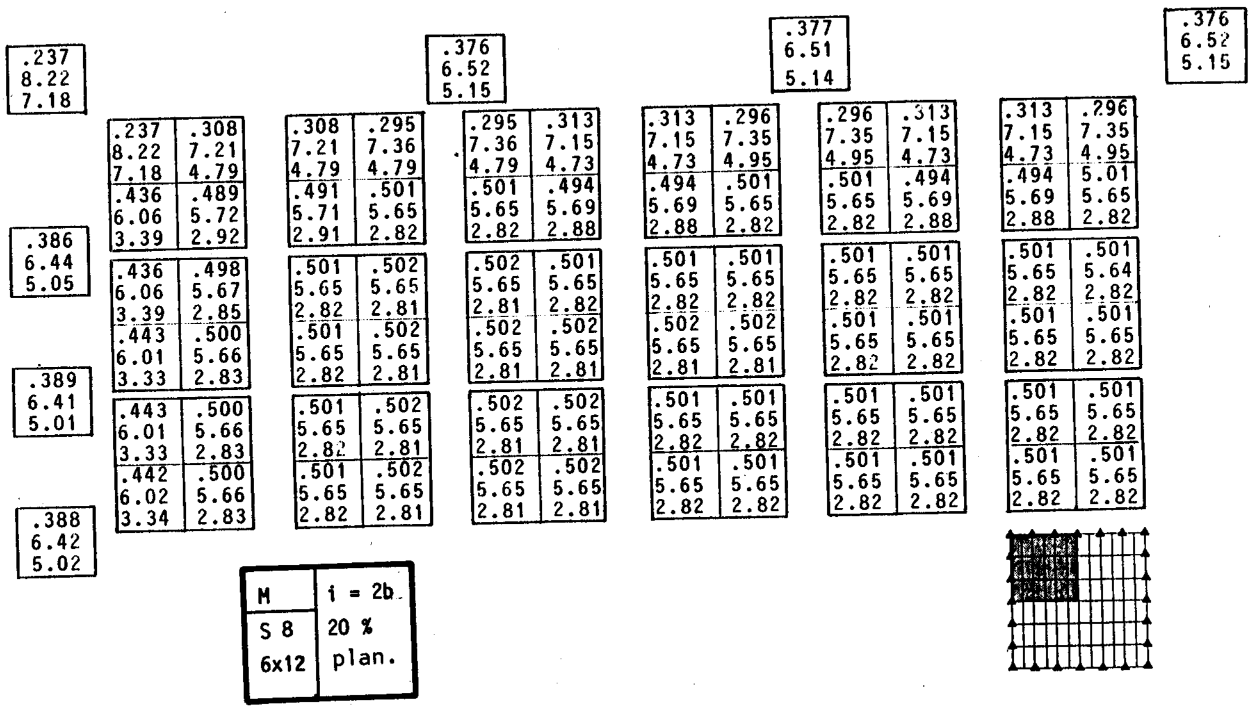
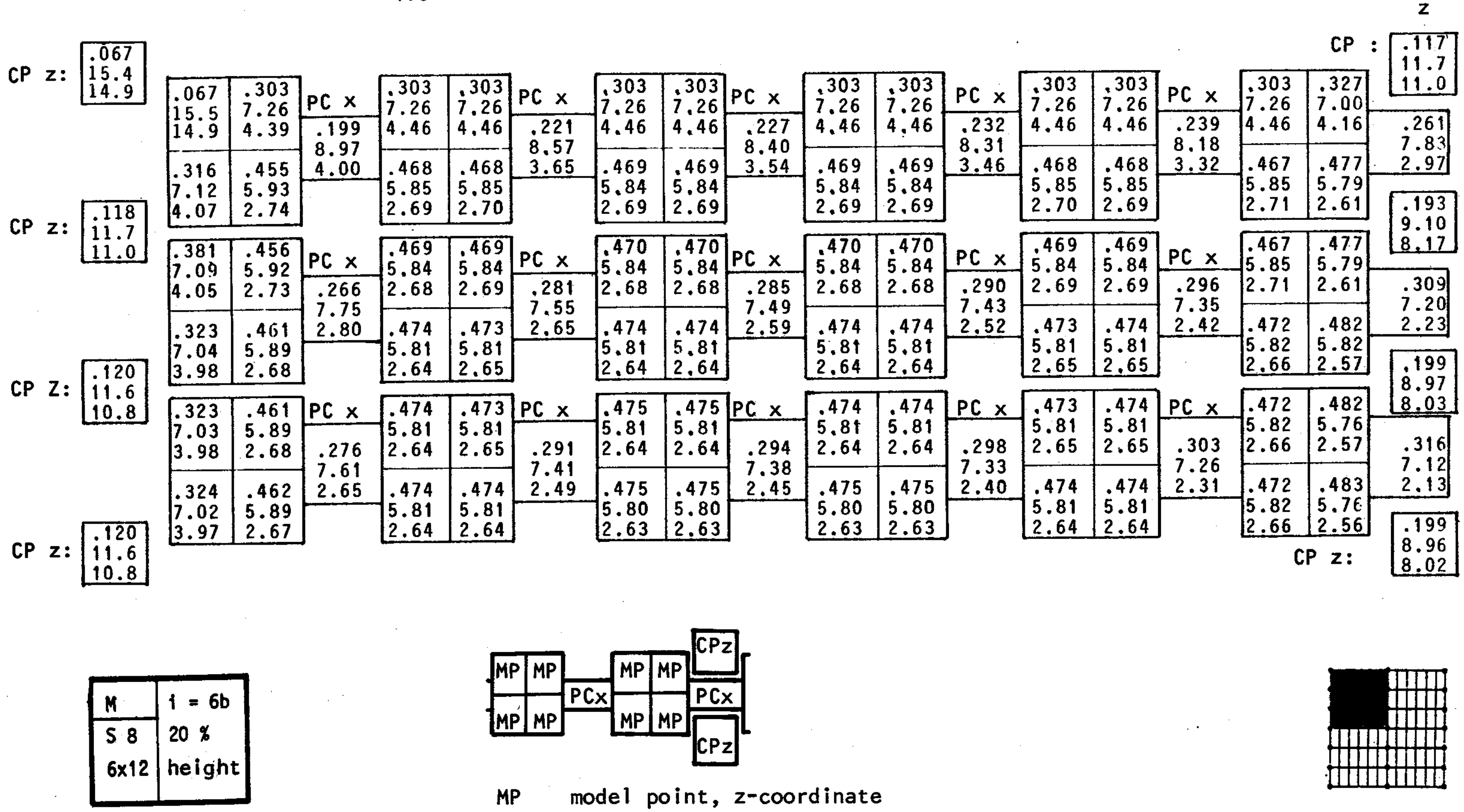


Fig. 7: Redundancy numbers r_i, controllability and sensitivity factors δ'_{oi} and $\bar{\delta}_{oi}$ of planimetric coordinates of a block with 6 x 12 = 72 independent models with sparse and dense control; 4 pairs of tie points in the corners of each model; values for x and y identical

Fig. 8: Redundancy numbers r_i, controllability and sensitivity factors δ'_{0i} and $\bar{\delta}_{0i}$ of height coordinates and of x-coordinates of projection centres of a block with 6 x 12 = 72 models, control point interval i = 6 b; 4 pairs of tie points in the corners and projection centres (x, y and z) per model.

|| Comment || : The values for the y- and z-coordinates of the projection centres are nearly independent of the location within the block.

y:	0.44	z:	0.40
	6.0		6.3
	1.6		0.3



MP model point, z-coordinate
PCx projection centre, x-coordinate
CPz control point, z-coordinate

B	i = 6b
S 18	20 %
6x13	x

CP (left):

x	y	z
.058 16.6 16.1	.050 17.9 17.4	.054 17.3 16.8

CP (right):

x	y	z
.160 10.0 9.18	.156 10.1 9.29	.010 12.7 12.0

A1	A2	B1	B2	B3	C1	C2	D1	D2	D3	E1	E2
.134 10.9 8.69	.122 11.4 8.79	.133 11.0 8.74	.489 5.72 3.51	.123 11.4 9.18	.122 11.4 9.20	.490 5.71 3.50	.122 11.5 9.22	.489 5.72 3.51	.124 11.4 9.13	.124 11.4 9.13	.487 5.73 3.53
.000	.145 10.5 9.02	.000	.579 5.26 3.01	.146 10.5 9.05	.145 10.5 9.10	.584 5.23 2.98				.147 10.4 9.02	.586 5.23 2.96
.469 5.84 3.18	.467 5.86 2.90	.435 6.07 3.51	.625 5.06 2.48	.488 5.73 3.03	.490 5.71 3.01	.647 4.97 2.33				.490 5.72 3.02	.648 4.97 2.32

CP z: .087 13.6 13.0 (left); CP z: .156 10.1 9.31 (right)

A1	A2	B1	B2	B3	C1	C2	E1	E2
.471 5.83 3.15	.469 5.84 2.88	.437 6.05 3.49	.631 5.03 2.44	.481 5.71 3.01	.494 5.69 2.98	.653 4.95 2.28	.493 5.70 2.99	.648 4.97 2.32
.000	.147 10.4 8.94	.000	.488 5.22 2.95	.149 10.4 8.93	.147 10.4 9.02	.597 5.18 2.89	.149 10.4 8.93	.597 5.18 2.88
.473 5.82 3.14	.469 5.84 2.88	.439 6.04 3.47	.635 5.02 2.41	.492 5.70 3.00	.495 5.68 2.97	.656 4.94 2.26		

CP z: .094 13.1 12.4 (left); CP z: .169 9.72 8.86 (right)

A1	A2	B1	B2	B3	C1	C2	D1	D2	E1	E2
.474 5.81 3.13	.470 5.84 2.88	.441 6.02 3.45	.635 5.02 2.41	.493 5.70 2.99	.495 5.68 2.97	.656 4.94 2.26				
.000	.147 10.4 8.93	.000	.589 5.21 2.94	.149 10.4 8.93	.147 10.4 9.00	.597 5.18 2.88	.149 10.4 8.93	.597 5.18 2.88	.149 10.4 8.93	.597 5.18 2.88
.498 5.67 2.93	.470 5.83 2.87	.444 6.01 3.42	.641 5.00 2.37	.493 5.70 2.99	.495 5.69 2.97	.657 4.93 2.25	.496 5.68 2.97	.656 4.94 2.36	.494 5.69 2.98	.653 4.95 2.28

CP (left):

.140 10.7 9.89	.166 9.83 8.98	.105 12.3 11.7

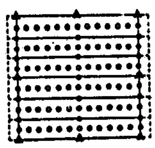
CP z: .189 9.21 8.30 (right)

B	i = 6b
S 18	20 %
6x13	y

A1	A2	B1	B2	B3	C1	C2	D1	D2	D3	E1	E2
.306 7.23 4.17	.378 6.50 4.20	.328 6.98 4.64	.447 5.98 3.99	.421 6.17 3.64	.420 6.17 3.64	.488 5.72 3.64	.424 6.14 3.61	.495 5.68 3.59	.426 6.13 3.59	.433 6.08 3.52	.538 5.45 3.25
.366 6.61 4.12	.518 5.56 3.20	.366 6.61 4.67	.536 5.46 3.33	.525 5.52 3.22	.518 5.56 3.27	.544 5.42 3.27				.525 5.52 3.22	.555 5.37 3.19
.327 7.00 3.91	.462 5.88 3.39	.370 6.58 4.16	.487 5.73 3.65	.492 5.70 3.00	.490 5.71 3.01	5.10 5.60 3.46				.502 5.65 2.91	.451 5.44 3.23

A1	A2	B1	B2	B3	C1	C2	E1	E2
.331 6.96 3.87	.469 5.84 3.33	.374 6.54 4.10	.515 5.57 3.43	.511 5.60 2.84	.512 5.59 2.83	.548 5.41 3.18	.519 5.55 2.78	.567 5.31 3.04
.377 6.51 4.01	.533 5.48 3.08	.377 6.51 4.55	.571 5.29 3.07	.541 5.44 3.10	.531 5.49 3.10	.581 5.25 3.00	.539 5.45 3.12	.582 5.24 2.99
.331 6.95 3.86	.470 5.83 3.33	.376 6.52 4.09	.515 5.57 3.43	.510 5.60 2.85	.512 5.59 2.83	.544 5.43 3.20		

A1	A2	B1	B2	B3	C1	C2	D1	D2	E1	E2
.333 6.94 3.84	.470 5.84 3.33	.367 6.52 4.08	.518 5.56 3.41	.511 5.60 2.84	.512 5.59 2.83	.549 5.40 3.17				
.379 6.50 3.99	.534 5.47 3.07	.379 6.50 4.53	4.75 5.28 3.05	.543 5.43 3.09	.534 5.48 3.16	.581 5.25 3.00	.540 5.44 3.11	.583 5.24 2.99	.539 5.45 3.11	.583 5.24 2.99
.333 6.93 3.84	.477 5.79 3.27	.401 6.31 3.82	.520 5.55 3.39	.514 5.58 2.81	.514 5.58 2.82	.550 5.40 3.16	.518 5.56 3.12	.556 5.37 2.78	.520 5.55 2.77	.569 5.30 3.02



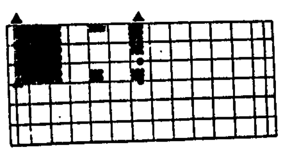


Fig. 9: Redundancy numbers r_i, controllability and sensitivity factors δ'_{0i} and $\bar{\delta}_{0i}$ of x- and y-coordinates of a bundle block with 6 x 13 = 84 images; point interval i = 6 b; 9 pairs of tie points at the standard positions

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lap and that the controllability of the control points also depends on the control point interval i/b.

3.5 Fig. 10 summarizes the main results of the investigation with regard to the reliability of the photogrammetric tie points. Here the maximum controllability factors δ'_{oi} and the maximum sensitivity factors $\bar{\delta}_{oi}$ are given for the three areas of interest, the values for the corners given separately.

The controllability of model coordinates obviously is increased using more tie points, especially at the corner and the border parts. Changing from 4 tie points (S 4) to double points (S 8) already leads to an acceptable reliability of the coordinates ($\bar{\delta}_{oi} \leq 4$). Double blocks cannot really be made more reliable by increasing the tie point density.

The situation is different for bundle blocks. Using more tie points (S 9 → S 18) does not increase the reliability, due to the already mentioned points in the middle of the strips, where additional points do not change the weak local geometry. The reliability of bundle blocks only can be improved by using higher coverage, i.e. 60 % sidelap or two single blocks flown crosswise.

3.6 The controllability of horizontal and vertical control points is given in Figs. 11 and 12 for single blocks. Double blocks will usually be applied in special cases in which high precision is demanded and the reliability of the control points will (hopefully) be guaranteed by geodetic means. Table 2 also contains approximations for the values of control points in the other areas of the blocks. The sensitivity values, which are not shown, can easily be obtained from $\bar{\delta}^2_{oi} = \delta'^2_{oi} - \delta^2_o$ and in most cases $\bar{\delta}_{oi} \cong \delta'_{oi}$ as $\delta_o << \delta'_{oi}$. The dependency of the reliability on the control point interval i/b is different for horizontal and vertical control points. The factors δ'_{oi} of horizontal control points increase approximately proportionally to the control point interval, whereas the controllability of vertical control points increases only with the square root of the interval i/b. This can be explained by the different control point pattern: single horizontal control points at the perimeter versus chains of vertical control points across the block. The absolute values are rather high ($\delta'_{oi} > 10$) even for small distances i/b.

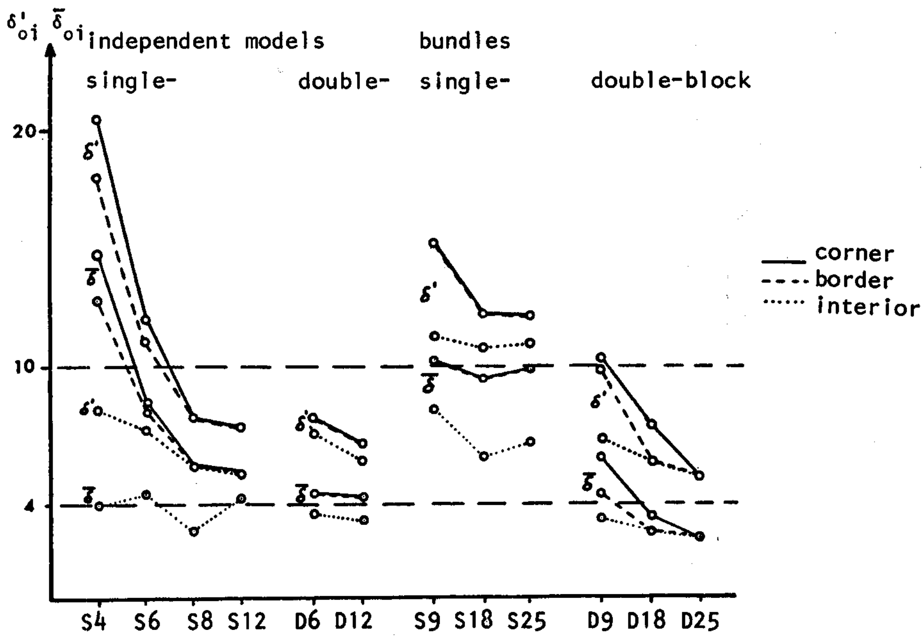

Fig. 10: Maximum values δ'_{oi} and $\bar{\delta}_{oi}$ of controllability and sensitivity of photogrammetric blocks

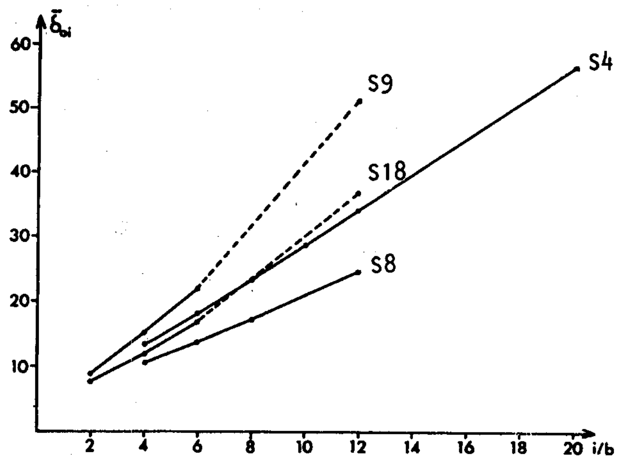

Fig. 11: Controllability factors δ'_{oi} of horizontal control points, single blocks, corners[1]

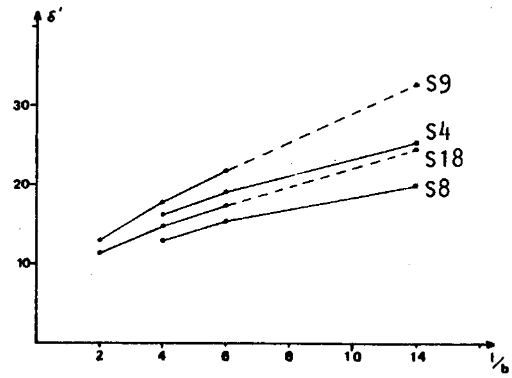

Fig. 12: Controllability factors δ'_{oi} of vertical control points, single blocks, corners[1]

[1]The end points of the dashed lines result from blocks with only 4 horizontal control points and 2 chains of vertical control points

Table 2: Controllability factors δ'_{oi} of horizontal and vertical control points in dependency of the control point distance i in blocks with 20 % sidelap (δ_o = 4)

block type	tie points	position of CP within block	$(\delta'_{oi})^2$ horiz. CP	$(\delta'_{oi})^2$ vertic. CP
independent models	4/model	corner	64 + 8 $(i/b)^2$	50 + 51 (i/b)
		border	56 + 1.9 $(i/b)^2$	37 + 27 (i/b)
		interior		19 + 14 (i/b)
	2x4/model	corner	64 + 4 $(i/b)^2$	58 + 30 (i/b)
		border	40 + 1.2 $(i/b)^2$	38 + 16 (i/b)
		interior		24 + 10 (i/b)
bundles	9/image	corner	42 +12.8 $(i/b)^2$	80 (i/b)
		border	30 + 3.7 $(i/b)^2$	42 (i/b)
		interior		21 (i/b)
	2x9/image	corner	48 + 6.9 $(i/b)^2$	42 + 42 (i/b)
		border	30 + 2.1 $(i/b)^2$	22 + 22 (i/b)
		interior		11 + 11 (i/b)

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Schriftenreihe, Heft 8, 1982

4. Conclusions

4.1 Photogrammetric point determination can reach a high reliability. Summarizing, this is the conclusion which can be drawn from this investigation. The stable geometry of photogrammetric blocks is the reason for good experiences gained in practical application. The results, however, show the weak areas in photogrammetric blocks: the geodetic control, the perimeter of the blocks and the points with only few rays in bundle blocks.

The main result for project planning is the independency of the reliability on the block size and the block form and the moderate influence of the different block parameters on each other. This allows a separate discussion, especially of photogrammetric and geodetic observations. But also two different types of application have to be distinguished: the determination of pass points for a subsequent mapping and the densification of highly accurate point fields.

4.2 Block triangulation as a basis for mapping needs only 20 % sidelap. As only pass points in the edges of the images are needed also the bundle method can be used. The points in the middle of the strips then are not used after the adjustment any more. Double points are recommendable in any case. This not only increases the reliability, while not requiring much additional effort for targeting or point transfer and for measuring. But what is more important, this remedy also simplifies the error detection procedure, since the elimination of points does not weaken the connection. In blocks with independent models 4 pairs of tie points are sufficient. If, however, self calibration is applied, also tie points in the middle of the strips are necessary to guarantee the determinability of the additional parameters. Here single points suffice.

There are several possibilities to strengthen the border parts of the blocks:

- Increase of the tie point density at the perimeter of the block. Especially in independent model blocks this is a very effective action.
- Increase of the block size by one strip or 2 base lengths in strip direction, in order to keep the area of interest within the interior of the block, i.e. one base length at the perimeter is not used for mapping.
- Bordering the block by a strip, which strengthens the perimeter. This is a variant of the previous remedy against the weak geometry.

In all cases a high reliability can be obtained with sensitivity factors $\bar{\delta}_{0i} \leq 3$, which guarantee the quality of the result.

4.3 In contrast to mapping application aerotriangulation for purposes of photogrammetric network densification requires 4 fold overlap. With regard to reliability 60 % sidelap and cross flights are equivalent; cross flights, however, have some advantages in compensating systematic errors. Because of the high overlap each point can at least be measured in 4 images, which guarantees a location of gross errors. Thus no points are lost by the elimination of a single observation. Here the increase of the block by one base length is best for strengthening the border. Fig. 13 shows that in all cases sensitivity values of $\bar{\delta}_{0i} \lessapprox 4$ are achieved, even with single tie points. This makes photogrammetric point determination comparable to geodetic densification, if not superior.

4.4 The reliability of the control points will always cause problems. Even for small control point intervals only controllability values around $\delta_{0i}' \approx 10$ are reached. Thus it seems not to be possible to check the coordinates of the ground control during the adjustment. This check has to be done and documented by the geodesist.

Then only the targeting has to be kept under control. Groups of control points are excellent for this purpose, too. The distribution of the points within the groups may consider the following recommendations:

- The points should be determined as independently as possible and therefore may be laid wide apart.
- The points should belong to at least 2 models or 3 images in order to be able to distinguish photogrammetric and geodetic errors. Thus one should avoid to use the point in the corner of the block as control point, but rather use some tie points at the border possibly together with a point more inside the block.

B	i = 2b
D9 m.R.	60 %
4x9	x

CP:

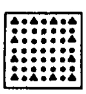
x	y	z
.336 6.90 5.62	.336 6.90 5.62	.181 9.40 8.50

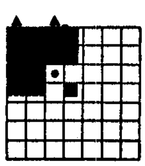
CP:

x	y	z
.547 5.41 3.61	.548 5.40 3.63	.300 7.31 6.11

	.000 ∞ ∞	.087 13.6 9.11	.000 ∞ ∞	.347 6.79 4.21	.087 13.6 9.13	.087 13.6 9.10	.346 6.80 4.22	.092 13.2 8.78	.087 13.6 9.13	.369 6.58 3.96
.199 8.98 6.89	.377 6.51 4.31	.350 6.76 4.47	.371 6.56 4.23	.593 5.19 2.44	.411 6.24 3.82	.387 6.43 4.06	.605 5.14 2.36	.388 6.42 4.06	.423 6.15 3.71	.609 5.13 2.33
.172 9.64 5.82	.473 5.81 2.95	.301 7.29 3.54	.359 6.68 2.82	.571 5.29 2.12	.350 6.76 2.92	.366 6.61 2.74	.580 5.25 2.04	.363 6.64 2.77	.369 6.58 2.70	.583 5.24 2.02
.199 8.98 5.22	.435 6.07 3.31	.301 7.30 3.55	.369 6.59 2.70	.548 5.40 2.31	.371 6.57 2.67	.345 6.81 2.98	.576 5.27 2.08	.346 6.80 2.97	.379 6.49 2.58	.558 5.36 2.23
.337 6.89 4.62	.515 5.57 3.05	.475 5.80 3.24	.491 5.71 3.11	.670 4.89 1.86	.510 5.60 2.95	.516 5.57 2.90	.681 4.85 1.77	.516 5.57 2.90	.521 5.54 2.86	.686 4.83 1.73
.237 8.22 4.51	.505 5.63 2.68	.350 6.76 2.92	.395 6.37 2.40	.589 5.21 1.97	.398 6.34 2.37	.391 6.40 2.44	.596 5.18 1.91	.391 6.40 2.44	.402 6.31 2.32	.595 5.19 1.92
.223 8.46 4.74	.481 5.77 2.89	.347 6.79 2.96	.382 6.48 2.55	.587 5.22 1.98	.385 6.44 2.51					
.377 6.52 4.18	.584 5.24 2.52	.515 5.57 2.91	.553 5.38 2.60	.681 4.85 1.78	.558 5.36 2.56	CP z:	.463 5.88 4.31			
.235 8.25 4.54	.486 5.74 2.84	.350 6.76 2.92	.385 6.45 2.52	.590 5.21 1.96	.388 6.42 2.48					
.240 8.16 4.44	.509 5.61 2.65	.358 6.69 2.83	.397 6.34 2.37	.592 5.20 1.94	.399 6.33 2.35				.403 6.30 2.31	.597 5.18 1.90
.373 6.55 4.22	.524 5.52 2.98	.493 5.70 3.09	.505 5.63 2.99	.677 4.86 1.80	.524 5.52 2.83				.533 5.48 2.76	.690 4.81 1.70

B	i = 2b
D18 m.R.	60 %
4x9	x





CP:

x	y	z
.399 6.34 4.91	.399 6.34 4.91	.228 8.37 7.35

CP:

x	y	z
.602 5.16 3.25	.595 5.19 3.30	.364 6.63 5.28

	.000 ∞ ∞	.126 11.3 9.05	.000 ∞ ∞	.502 5.64 3.40	.126 11.3 9.03	.126 11.3 9.05	.504 5.63 3.39	.128 11.2 8.94	.126 11.3 9.03	.512 5.59 3.32
.348 6.78 5.01	.542 5.44 3.29	.501 5.65 3.54	.524 5.52 3.36	.711 4.74 2.10	.555 5.37 3.12	.519 5.55 3.40	.725 4.70 2.00	.519 5.55 3.40	.562 5.34 3.07	.719 4.72 2.05
.417 6.19 3.67	.429 5.04 2.45	.553 5.38 2.50	.571 5.29 2.35	.729 4.69 1.73	.577 5.27 2.31	.584 5.23 2.25	.733 4.67 1.70	.584 5.24 2.25	.586 5.23 2.23	.734 4.67 1.69
.348 6.78 4.40	.571 5.29 2.87	.473 5.81 3.16	.523 5.53 2.74	.689 4.82 2.02	.532 5.48 2.67	.497 5.68 2.96	.713 4.74 1.85	.497 5.68 2.96	.540 5.44 2.60	.694 4.80 1.99
.500 5.66 3.55	.649 4.97 2.53	.639 5.00 2.52	.638 5.01 2.53	.777 4.54 1.64	.656 4.94 2.41	.660 4.92 2.38	.784 4.52 1.59	.660 4.92 2.38	.662 4.92 2.37	.786 4.51 1.57
.448 5.98 3.39	.660 4.92 2.23	.575 5.27 2.32	.595 5.19 2.16	.737 4.66 1.67	.601 5.16 2.11	.597 5.18 2.14	.741 4.65 1.64	.597 5.18 2.14	.605 5.14 2.08	.740 4.65 1.65
.443 6.01 3.43	.633 5.03 2.43	.575 5.28 2.32	.582 5.24 2.27	.736 4.66 1.68	.594 5.19 2.17					
.519 5.55 3.40	.703 4.77 2.16	.655 4.94 2.41	.680 4.85 2.24	.783 4.52 1.60	.685 4.83 2.21	CP z:	.537 5.46 3.72			
.448 5.98 3.38	.636 5.02 2.41	.576 5.27 2.31	.584 5.24 2.25	.738 4.66 1.67	.595 5.19 2.16					
.450 5.97 3.37	.664 4.91 2.21	.579 5.26 2.29	.598 5.17 2.14	.738 4.66 1.66	.603 5.15 2.10				.606 5.14 2.07	.742 4.64 1.64
.518 5.56 3.40	.645 4.95 2.49	.648 4.97 2.46	.645 4.98 2.48	.782 4.52 1.60	.664 4.91 2.35				.668 4.89 2.32	.789 4.50 1.55

Fig. 13: Redundancy numbers r_i, controllability and sensitivity factors δ'_{oi} and $\bar{\delta}_{oi}$ for the x-coordinates of the image points and for the x-, y- and z-coordinates of the control points in a bundle block with 49 images; sidelap 60 %, control point interval i = 2 base lengths, 9 tie points and 9 pairs of tie points per image. The values for the y-coordinates are achieved by mirroring the values at the main diagonal

- At least 3 points should be targeted, in order to be able to control the targeting by checking the similarity of triangles, thus to be independent of the local scale. If remeasurements are not possible or not wanted, one should use at least 4 points. Then a single gross error can be located without additional information. In case the distances between the points within a group are small, the controllability is higher (due to the smaller influence of the local scale). Then one needs one point less, i.e. 2 or 3 points at least, provided the points are rather independent.

Using groups of points one must realize that a joint shift of the group is more difficult to detect than if one would use a single point, due to the higher weight of the group.

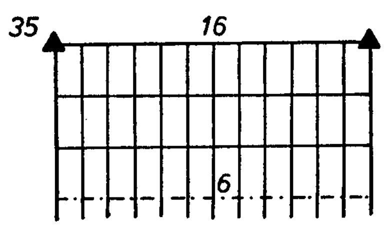
The joint effect of bordering, using double tie points and groups of control points, on the reliability is shown in Fig. 14, summarizing the result of this investigation. The completion of the expedients, which are necessary to reach a high precision any way, can lead to a high reliability of aerial triangulation.

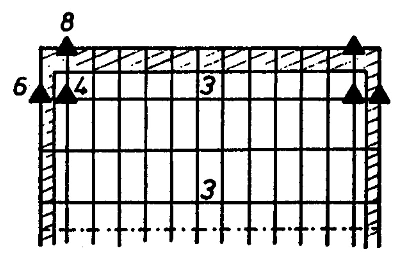
Fig. 14: Optimization of the reliability of a planimetric block with independent models, 4 tie points per model

a) given block, external reliability at control point, at border and interior of block;
$\bar{\delta}_{max} = 35$

b) optimized block, external reliability at control points, border and interior of block
- using groups of control points
- using pairs of tie points
- not using the hatched parts of the block;

$\bar{\delta}_{max} = 4$





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Institut für Photogrammetrie
Schriftenreihe, Heft 8, 1982

Abstract

The theory of reliability treats the ability to detect gross errors by using statistical tests and the sensitivity of the result with respect to non-detectable gross errors. In an extensive investigation the theory of reliability was applied to photogrammetric point determination. It results in clear recommendations for the project planning, consisting in the appropriate choice of the block parameters as overlap, point distribution of control points, which leads to a homogeneous precision as well as to a high reliability of block triangulation.

Zuverlässigkeit der Blocktriangulation

Zusammenfassung

Die Theorie der Zuverlässigkeit behandelt die Aufdeckbarkeit von groben Datenfehlern mit Hilfe statistischer Tests und die Empfindlichkeit der Ergebnisse gegen nicht erkennbare grobe Fehler. In einer umfangreichen Untersuchung wurde die Theorie der Zuverlässigkeit auf die photogrammetrische Punktbestimmung angewandt. Es ergaben sich klare Empfehlungen für die Wahl der Blockparameter wie Überdeckung, Punktdichte und Paßpunktbesetzung, sodaß neben einer gleichmäßigen Genauigkeit auch eine hohe Zuverlässigkeit der Blocktriangulation erreicht werden kann.

Fiabilitê de l'aêrotriangulation par blocs

Rêsumê

La thêorie de la fiabilitê traite la dêtection de grandes erreurs au moyen de tests statistiques et la sensibilitê des rêsultats aux grandes erreurs qui n'ont pas êtê relevêes. La thêorie de la fiabilitê a êtê appliquêe à la dêtermination photogrammêtrique de points dans le cadre d'une investigation approfondie. Le choix de paramètres du bloc a êtê recommandê, tels que recouvrement, densitê des points et rêpartition des points d'appui permettant d'obtenir non seulement une prêcision homogêne mais encore une grande fiabilitê de la triangulation par blocs.

Fiabilidad de la triangulaciôn por bloques

Resumen

La teorîa de la fiabilidad trata la posibilidad de detectar graves errores de datos recurriendo a ensayos estadîsticos asî como la sensibilidad de los resultados frente a errores graves no detectados. Durante unas investigaciones muy extendidas, se ha aplicado la teorîa de la fiabilidad a la determinaciôn fotogramêtrica de puntos. El resultado eran recomendaciones muy claras en cuanto a la eleccion de los parâmetros de bloques, tales como recubrimiento y densidad de los puntos de apoyo, de modo que pueda conseguirse, además de una densidad homogênea, tambiên alta fiabilidad de la triangulaciôn por bloques.

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