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NEW RESULTS OF BUNDLE BLOCK ADJUSTMENT WITH ADDITIONAL PARAMETERS

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The method of bundle adjustment

The most rigorous mathematical model of aerial triangulation is based on the principle of the bundle method. On the basis of measured and reduced image coordinates and the control point coordinates in the terrain system, the parameters of exterior orientation and the unknown ground coordinates of the measured points are determined by simultaneous adjustment. The functional relationship is provided by the perspective conditions between measured image and unknown ground coordinates.

The theoretically expected accuracy of block triangulation has essentially been found confirmed in practice. However, closer analysis of the results reveals significant discrepancies between these and theoretical expectations (see |1| and |3|). This is due to the simplified assumption of exclusively random errors of image coordinates. A refined error model which also takes into account systematic errors is therefore required. Parameters for possible systematic errors are introduced and their values determined in block adjustment from the given data material. These methods have become known as self-calibrating techniques. However, the unknown parameters are introduced as observations with a certain weight and a corresponding, additional error equation in order to avoid problems of the condition of the equation system. This at the same time allows a great margin of freedom regarding the type and number of additional parameters.

The set (by EBNER) with 12 parameters (see |4|) proved to be the one that was easiest to realize both numerically and statistically. This corrects all systematic deformations at the nine schematic points of a photo. Together with the six parameters of exterior orientation, the 12 parameters form an orthogonal system, provided that measured image points are of regular distribution. This allows independent statistical analysis of individual parameters after their determination. A set with 44 parameters referring to 25 schematic points (by GRÜN) is possible if the image points in the data material are suitably distributed (see |7|).

The PAT-B program

As early as 1971, the Institute of Photogrammetry of Stuttgart University started developing a bundle program. At the time, however, the program was suitable only for small theoretical studies. During the past two years, it has been considerably improved with respect to computer time, generality and operational aspects. Early this year, self calibration was added, and by the end of this year, the program will be operational, similar to PAT-M43.

The program is written in FORTRAN IV and requires an internal memory of at least 24 K 16-bit words. In smaller computers, it will be subject to certain limitations regarding block size, the number of points and the number of additional parameters for self calibration.

If the initial values for exterior orientation are given, approximate values can be computed for the ground coordinates. Should the initial values be unknown, these also may be computed.

The linearized error equations reduced to the corrections of orientation parameters and parameters of self calibration are formed directly and solved by a modified Cholesky method. The corrected ground coordinates are computed after every iteration step. Together with the corrected orientation parameters, these are used as approximate values for the next iteration step.

If the rms corrections at the image coordinates are less than 500 µm, the parameters selected for self calibration are introduced into the adjustment.

The additional parameters can be selected from a set with 12 or another one with 44 parameters, depending on the number and location of image points. The coefficients of the parameters are normalized so that parameter size directly gives the maximum correction in microns caused by the parameter in the image. The different parameters may be applied for all photos or for groups of them. It is thus possible to recognize and correct even local systematics. For operational reasons the parameters can be introduced with a very small standard deviation. It is thus possible to recognize and analyze systematics without correcting them.

After adjustment, the different parameters are tested for determinability and statistical significance with the following formulas (see |6|).

Determinability:
$$\nabla_o t_k^{(o)} = \sigma_{t_k} \, \delta_o \sqrt{\frac{1 - r_k}{r_k}}$$

Statistical significance:
$$\omega_k^{(o)} = \frac{\hat{t}_k}{\sigma_{t_k} \sqrt{r_k} \sqrt{1 - r_k}}$$

where

σ_{t_k} = rms error before adjustment

δ_o = statistical parameter

r_k = redundancy component

$\hat{t}_k$ = estimated parameter

The Appenweier project

In 1973, the controlled Appenweier test for the densification of a given third-order trigonometric network was performed on the initiative of the Landesvermessungsamt Baden-Württemberg.

The Appenweier area is located in the Baden region, in the upper Rhine plains. It covers a surface of 9.1 km x 10.4 km = 95 km^2 and contains 24 given trigonometric points of second and third order. In order to obtain a sufficient number of vertical control points, the elevation of additional 38 tie points was also observed. Moreover, the coordinates of 85 points of roughly regular distribution were known (see Figure 1). All control points were signalized, the tie points were marked by triple signals for the nine schematic points of a model (see |2|).

The photo flight was done by the Photogrammetric Division of Rheinische Braunkohle AG with a Zeiss RMK A 15/23 wide-angle camera in the spring of 1973.

The test was designed to get a planimetric accuracy of 3 cm. However, instead of a single flight at 1:4000 scale, a four-fold flight at 1:7800 scale was made (with crossed and anti-parallel flight axes), of which an increase in accuracy by more than the factor 2 as compared with a single flight at the same scale was expected in simultaneous block adjustment.

The 478 film diapositives of the aerial photography were measured on a Zeiss PK-1 Monocomparator at the Institute of Photogrammetry of Stuttgart University.

Distortion, earth curvature and refraction were corrected within the fiducial-mark system during reduction of the image coordinates to the principal point.

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Outline of block adjustment

The data obtained in the Appenweier test project allow the following points to be studied:

1. Improvement of absolute accuracy and maximum error by the introduction of additional parameters for the correction of systematic errors into block adjustment.
2. Analysis of systematic image errors for block-invariant and local systematics and the effect of the corresponding corrections.
3. The effect on absolute accuracy of the selection of parameters for determinability and statistical significance.
4. The effect of various overlap configurations (single block, double block, four-fold block) on absolute accuracy.
5. Two control versions: |1| Use of all given horizontal trigonometric points (perimeter control + 8 points in the interior of the block), |2| Horizontal points only at the perimeter of the block at intervals of about two base lengths (trigonometric points + 8 additional points from the reference points).

Due to the location of the tie points near the nine schematic points of the photo, the set with 12 parameters was chosen for self calibration.

All parameters with $\nabla_0 t_k > 4.0$ were omitted.

The parameters with $\omega_k > 4$ were determined during adjustment and the systematics were corrected, while for the parameters with $4 > \omega_k > 3.3$ only the statistical tests were performed without any correction for systematics.

The zero hypothesis chosen for determinability and statistical significance was the four-fold block with all control points for selection I and the double block with dense horizontal perimeter control for selection II.

Results of block adjustment

1. If we compare the absolute accuracy of the single and double blocks without and with additional parameters (Tables 1 and 2), it is obvious that accuracy was improved from 5.7 cm to 3.4 cm and from 3.4 cm to 2.0 cm (by a factor of 1.7 in each case). In the given conditions, even the double block gave absolute accuracy only slightly above the accuracy of the control points used. This goes to prove that bundle block adjustment with additional parameters as the theoretically most rigorous method fully satisfies the high expectations regarding accuracy.
2. A comparison with previous adjustments by the method of independent models (see |2|) shows - although different measurements were used in that case - that the adjustment by the bundle method without additional parameters is less favorable in the single block than by the method with independent models ($\mu_{x,y}$ = 5.1 cm).

   However, the effectiveness of the additional parameters in the method of independent models is less or, in other words, it is more difficult to control systematics with simple means ($\mu_{x,y}$ = 4.4 cm in SN block, factor of increase only 1.4).

   This also means, however, that a correction of systematic errors is absolutely necessary in bundle block adjustment. The single block with self calibration already shows the same accuracy as the double block without self calibration.
3. Block adjustment with all trigonometric control points (Table 3) - likewise 27, but distributed more or less evenly over the entire area - produced the same absolute accuracy both for the single and the double

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blocks. Assuming only random errors, absolute accuracy with the same number of control points - but located along the perimeter - should be higher, since the horizontal control points in the interior have no effect on absolute accuracy. On the other hand, the control points in the interior of the block result in better determinability of the systematics in block adjustment with self calibration. These two effects are about the same, at least for this size of block.

4. When selecting the parameters, allowance should be made both for the determinability and the statistical significance of the different parameters, determinability clearly having priority. Non-singular parameters may give less good results, since the magnitude of the parameters depends on random effects instead of on the systematics of the block. The lower the redundancy and overlap, the less favorable becomes the determinability of the different parameters. This reveals a weakness of self calibration: existing systematics can only be eliminated if there is sufficient redundancy. Using all 12 parameters, the single blocks give slightly less favorable results than a selection by the criteria of determinability and significance (Table 4). The double blocks, however, are not less accurate, since here only some parameters cannot be determined (compare Table 5 with Table 6).

5. The assumption that the systematics would be identical for an entire block is only partly correct (see Figure 2). The results obtained with a block-invariant use of the parameters (Table 5) are less favorable both for the single blocks and the double blocks than with a strip-invariant use of the parameters: 3.8 cm and 2.8 cm versus 3.4 cm and 2.0 cm. However, it can be clearly seen that the largest part of the systematics is identical for the entire block (see Figure 3) but that it is covered up by noticeable local systematics. Above all in the double blocks, where local systematics can be easily determined by good geometry and particularly high redundancy, preference should be given to local use over a global one. In single blocks with less redundancy than in the Appenweier project, however, the global use of additional parameters at least should not result in lower accuracy.

   A clear scatter of σ_o is, however, evident when we compare the single blocks with the double blocks. This points to uncorrected residual systematics of photogrammetric measurements even in stripwise use.

6. Although it does not seem to correspond to the given accuracies, block adjustment with additional parameters and a standard deviation of $\sigma_{pp} = 0$ in the present case (Table 7) gives a slight improvement over the same adjustment with $\sigma_{pp} = 1.5$ cm (Table 6). We shall have to see whether this also applies to other overlap configurations and selections of parameters. It may be assumed that residual systematics at the control points will be eliminated in this variant. The additional parameters here act like a least-squares interpolation.

7. The ratio between the accuracy $\mu_{x,y}$ of single blocks and double blocks in the stripwise use of the parameters (Tables 2 and 3) is 1 : 0.59.

   Referred exclusively to random errors, the theoretical ratio should be smaller than $1 : 1/\sqrt{2} = 1 : 0.71$. Contrary to earlier adjustments (see |2|), the result fully meets the expectations. An essential increase in absolute accuracy by four-fold blocks is possible only where the accuracy of the control points (~ 1.5 cm - 1.8 cm) has not yet been attained (see Table 6).

8. In addition to a noticeable increase in absolute accuracy (rms values from 85 known reference points), the noticeable improvement of the maximum error in the adjustment with additional parameters likewise is of great importance. With the most effective combinations of parameters (Tables 2 and 3), the maximum error is only 12.3 cm or 13.8 cm for single blocks and goes down to about 6 cm with double blocks.

9. As has been mentioned before, the Appenweier project is unsuitable for further analysis of vertical accuracy in bundle adjustment with additional parameters. For such studies, the control point interval within the different control chains would have to be reduced to two base lengths. In spite of this, however, we may say that the additional parameters improve the absolute accuracy in single blocks (Tables 1, 2, 3) from 8.5 cm to 7.2 cm and that the maximum error can be reduced from 29.4 cm to 21.4 cm. In the double blocks, it was found that an absolute accuracy of 6.3 cm and a maximum error of 15.7 cm could not be improved.

   Theoretical investigations (see |5|) have shown, however, that further considerable increase in vertical accuracy cannot be expected from self calibration, since systematic image errors in bundle blocks do not have the same effect on vertical accuracy as on horizontal accuracy. This is also proved by a comparison with accuracies attained by the method of independent models (see |2|) in single blocks, double blocks and triple blocks where vertical accuracy is only 10.5 cm - 9.1 cm - 7.5 cm and the maximum error 30.3 cm - 23.2 cm - 15.7 cm. In bundle block adjustment with self calibration, the vertical accuracy of the four-fold block in the method of independent models is reached already in single blocks.

Summary

1. In bundle block adjustment, the correction of systematic errors is indispensable.
2. The systematic errors are largely block-invariant, but they are covered up by significant local systematics.
3. If there is sufficient redundancy, the local use of the parameters may also serve to correct local systematics and improve absolute accuracy.
4. The parameters should be chosen on the basis of determinability + statistical significance.
5. If the parameters are used by strips, absolute accuracy can be improved by a factor of 1.7 over an adjustment without parameters.
6. In accordance with theoretical expectation, double overlap considerably improves absolute accuracy.
7. Four-fold overlap could not noticeably increase absolute accuracy any further, since even the double block came close to control-point accuracy.
8. A correction of systematic errors is accompanied by a noticeable improvement of maximum errors.

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Table 1 APPENWEIER BUNDLE ADJUSTMENT WITHOUT SELF CALIBRATION
(wide-angle photography, photo scale 1:7800) σ_{pp} = 1.5 cm

Control points

Planimetry: dense perimeter control (~2 base lengths = 27 control points)

Elevation : 24 trigonometric points + 38 additional groups of three

Block	σ_o	μ_x	μ_y	$\mu_{x,y}$	μ_z	μ_x	μ_y	$\mu_{x,y}$	μ_z	$\varepsilon_{x,y\,max}$	$\varepsilon_{z\,max}$
	- µm -	- µm -				- cm -					
Single blocks (119 and 120 photos, respectively):											
EW	2.7	5.3	4.5	4.9	10.6	4.0	3.4	3.7	8.0	11.5	24.8
WE	3.0	6.6	5.2	6.0	12.4	5.0	3.9	4.5	9.3	14.8	28.9
NS	3.2	8.6	7.7	8.2	11.9	6.7	6.0	6.4	9.3	19.7	29.4
SN	3.3	9.6	9.1	9.4	9.6	7.4	7.1	7.3	7.4	42.9	29.2
Mean	3.0	7.7	6.9	7.3	11.2	5.9	5.3	5.7	8.5	42.9	29.4
Double blocks (239 photos):											
WE/NS	3.6	4.8	4.1	4.5	8.4	3.7	3.2	3.5	6.4	10.5	15.7
EW/SN	3.5	4.4	4.3	4.3	8.1	3.4	3.3	3.4	6.2	8.5	12.8
Mean	3.6	4.6	4.2	4.4	8.2	3.6	3.3	3.4	6.3	10.5	15.7

ε = maximum coordinate error at check points

μ_x, μ_y, μ_z = rms residual errors at check points,

$$\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$$

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Table 2 APPENWEIER BUNDLE ADJUSTMENT WITH SELF CALIBRATION - SELECTION out of 12 (parameters strip-invariant, wide-angle photography, photo scale 1:7800)

σ_{pp} = 1.5 cm

Control points

Planimetry: dense perimeter (~ 2 base lengths = 27 control points)
Height : 24 trigonometric points + 38 additional groups of three

Block	σ_o	μ_x	μ_y	$\mu_{x,y}$	μ_z	μ_x	μ_y	$\mu_{x,y}$	μ_z	$\varepsilon_{x,y\,max}$	$\varepsilon_{z\,max}$
	- µm -			µm -				- cm -			
Single blocks (119 and 120 photos, respectively):											
EW	2.1	3.7	3.3	3.5	9.3	2.8	2.5	2.6	7.0	8.2	20.6
WE	2.2	4.6	5.0	4.8	10.6	3.4	3.8	3.6	7.9	10.7	21.2
NS	2.4	4.4	3.3	3.9	8.7	3.4	2.6	3.0	6.8	9.4	21.1
SN	2.4	6.4	3.6	5.2	9.0	5.0	2.8	4.1	7.0	13.8	16.7
Mean	2.3	4.9	3.9	4.4	9.4	3.7	2.9	3.4	7.2	13.8	21.2
Double blocks (239 photos):											
WE/NS	2.5	2.9	2.9	2.9	8.5	2.2	2.2	2.2	6.5	7.0	18.0
EW/SN	2.6	2.6	2.0	2.3	7.6	2.0	1.5	1.8	5.8	4.8	13.6
Mean	2.6	2.7	2.5	2.6	8.1	2.1	1.9	2.0	6.2	7.0	18.0

ε = maximum coordinate error at check points

μ_x, μ_y, μ_z = rms residual errors at check points,

$$\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$$

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Table 3 APPENWEIER BUNDLE ADJUSTMENT WITH SELF CALIBRATION - SELECTION out of 12 (parameters strip-invariant, wide-angle photography, photo scale 1:7800)

σ_{pp} = 1.5 cm

Control points

Planimetry: trigonometric points (27 control points, well distributed)
Height : 24 trigonometric points + 38 additional groups of three

Block	σ_0	μ_x	μ_y	$\mu_{x,y}$	μ_z	μ_x	μ_y	$\mu_{x,y}$	μ_z	$\varepsilon_{x,y\,max}$	$\varepsilon_{z\,max}$
	- µm -		- µm -					- cm -			
Single blocks (119 and 120 photos, respectively):											
EW	2.0	3.7	3.7	3.7	9.4	2.8	2.8	2.8	7.1	7.8	20.4
WE	2.2	4.9	3.8	4.3	10.5	3.6	2.8	3.2	7.9	9.9	21.4
NS	2.4	4.9	3.9	4.4	8.7	3.8	3.1	3.5	6.8	10.4	20.0
SN	2.4	5.9	3.7	4.9	8.8	4.6	2.9	3.9	6.8	12.3	17.9
Mean	2.3	4.9	3.8	4.4	9.4	3.8	2.9	3.4	7.2	12.3	21.4
Double blocks (239 photos):											
WE/NS											
EW/SN	2.6	2.7	2.1	2.4	7.7	2.0	1.6	1.8	5.9	5.0	13.6
Mean											

ε = maximum coordinate error at check points

μ_x, μ_y, μ_z = rms residual errors at check points,

$$\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$$

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Table 4 APPENWEIER BUNDLE ADJUSTMENT WITH SELF CALIBRATION - 12 PARAMETERS (parameters strip-invariant, wide-angle photography, photo scale 1:7800)

σ_{pp} = 1.5 cm

Control points

Planimetry: dense perimeter (~ 2 base lengths = 27 control points)
Height : 24 trigonometric points + 38 additional groups of three

Block	σ_o	μ_x	μ_y	$\mu_{x,y}$	μ_z	μ_x	μ_y	$\mu_{x,y}$	μ_z	$\varepsilon_{x,y\,max}$	$\varepsilon_{z\,max}$
	- µm -		- µm -					- cm -			
Single blocks (119 and 120 photos, respectively):											
EW	2.0	3.9	4.3	4.1	9.5	2.9	3.2	3.1	7.1	9.5	17.0
WE	2.2	3.7	5.3	4.6	10.3	2.8	4.0	3.5	7.8	10.0	20.6
NS	2.3	5.3	5.0	5.1	8.9	4.1	3.9	4.0	7.0	12.8	18.2
SN	2.4	5.3	4.7	5.0	10.3	4.1	3.6	3.9	8.0	12.3	21.9
Mean	2.2	4.5	4.8	4.7	9.8	3.5	3.7	3.6	7.5	12.8	21.9

ε = maximum coordinate error at check points

μ_x, μ_y, μ_z = rms residual errors at check points,

$$\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$$

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Table 5 APPENWEIER BUNDLE ADJUSTMENT WITH SELF CALIBRATION - SELECTION out of 12 (parameters block-invariant, wide-angle photography, photo scale 1:7800) σ_{pp} = 1.5 cm

Control points

Planimetry: dense perimeter (~ 2 base lengths = 27 control points)
Height : 24 trigonometric points + 38 additional groups of three

Block	σ_o	μ_x	μ_y	$\mu_{x,y}$	μ_z	μ_x	μ_y	$\mu_{x,y}$	μ_z	$\varepsilon_{x,y}$ max	ε_z max
	- μm -	- μm -				- cm -					
Single blocks (119 and 120 photos, respectively):											
EW	2.1	4.7	4.0	4.4	10.5	3.6	3.0	3.3	7.9	9.2	22.1
WE	2.4	4.8	3.6	4.2	11.3	3.6	2.7	3.2	8.5	11.3	21.8
NS	2.5	6.3	3.6	5.1	9.3	4.9	2.9	4.0	7.3	12.5	17.8
SN	2.5	6.7	4.7	5.8	9.4	5.2	3.7	4.5	7.3	18.6	23.7
Mean	2.4	5.6	4.0	4.9	10.1	4.3	3.1	3.8	7.8	18.6	23.7
Double blocks (239 photos):											
WE/NS	2.7	3.7	2.7	3.2	8.2	2.8	2.1	2.5	6.3	7.2	17.0
EW/SN	2.7	3.6	3.1	3.4	8.5	2.8	2.4	2.6	6.5	9.3	16.4
Mean	2.7	3.7	2.9	3.3	8.3	2.8	2.3	2.6	6.4	9.3	17.0

ε = maximum coordinate error at check points

μ_x, μ_y, μ_z = rms residual errors at check points,

$$\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$$

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Table 6 APPENWEIER BUNDLE ADJUSTMENT WITH SELF CALIBRATION - 12 PARAMETERS (parameters block-invariant, wide-angle photography, photo scale 1:7800) σ_{pp} = 1.5 cm

Control points

Planimetry: dense perimeter (~ 2 base lengths = 27 control points)
Height : 24 trigonometric points + 38 additional groups of three

Block	σ_o	μ_x	μ_y	$\mu_{x,y}$	μ_z	μ_x	μ_y	$\mu_{x,y}$	μ_z	$\varepsilon_{x,y\,max}$	$\varepsilon_{z\,max}$
	- µm -		- µm -					- cm -			
Single blocks (119 and 120 photos, respectively):											
EW											
WE											
NS											
SN											
Mean											
Double blocks (239 photos):											
WE/NS	2.7	3.7	2.7	3.2	8.2	2.8	2.0	2.4	6.3	7.1	17.1
EW/SN	2.7	3.6	3.1	3.4	8.5	2.8	2.4	2.6	6.5	9.3	16.4
Mean	2.7	3.7	2.9	3.3	8.4	2.8	2.2	2.5	6.4	9.3	17.1
Four-fold block (478 photos):											
	2.8	3.4	2.5	3.0	7.9	2.6	1.9	2.3	6.1	7.2	15.4

ε = maximum coordinate error at check points

μ_x, μ_y, μ_z = rms residual errors at check points,

$$\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$$

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Table 7 APPENWEIER BUNDLE ADJUSTMENT WITH SELF CALIBRATION - 12 PARAMETERS (parameters block-invariant, wide-angle photography, photo scale 1:7800) σ_{pp} = 0 cm

Control points

Planimetry: dense perimeter (~2 base lengths = 27 control points)
Height : 24 trigonometric points + 38 additional groups of three

Block	σ_o	μ_x	μ_y	$\mu_{x,y}$	μ_z	μ_x	μ_y	$\mu_{x,y}$	μ_z	$\varepsilon_{x,y\,max}$	$\varepsilon_{z\,max}$
	- µm -		- µm -					- cm -			
Single blocks (119 and 120 photos, respectively):											
EW	2.2	4.6	4.0	4.3	10.6	3.5	3.0	3.3	8.0	8.9	22.2
WE	2.4	4.7	3.6	4.0	11.3	3.5	2.7	3.1	8.5	10.4	21.2
NS	2.5	5.9	3.6	4.9	9.4	4.6	2.8	3.8	7.4	12.7	18.8
SN	2.5	6.3	4.5	5.4	9.5	4.8	3.5	4.2	7.4	18.7	24.0
Mean	2.4	5.4	3.9	4.7	10.2	4.1	3.0	3.6	7.8	18.7	24.0
Double blocks (239 photos):											
WE/NS	2.7	3.4	2.5	3.0	8.3	2.6	1.9	2.3	6.4	6.6	17.1
EW/SN	2.7	3.6	3.1	3.3	8.2	2.7	2.3	2.5	6.3	9.9	16.6
Mean	2.7	3.5	2.8	3.2	8.3	2.7	2.1	2.4	6.4	9.9	17.1

ε = maximum coordinate error at check points

μ_x, μ_y, μ_z = rms residual errors at check points,

$$\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$$

Table 8 APPENWEIER BUNDLE BLOCK ADJUSTMENT
ABSOLUTE ACCURACY AND MAXIMUM ERROR

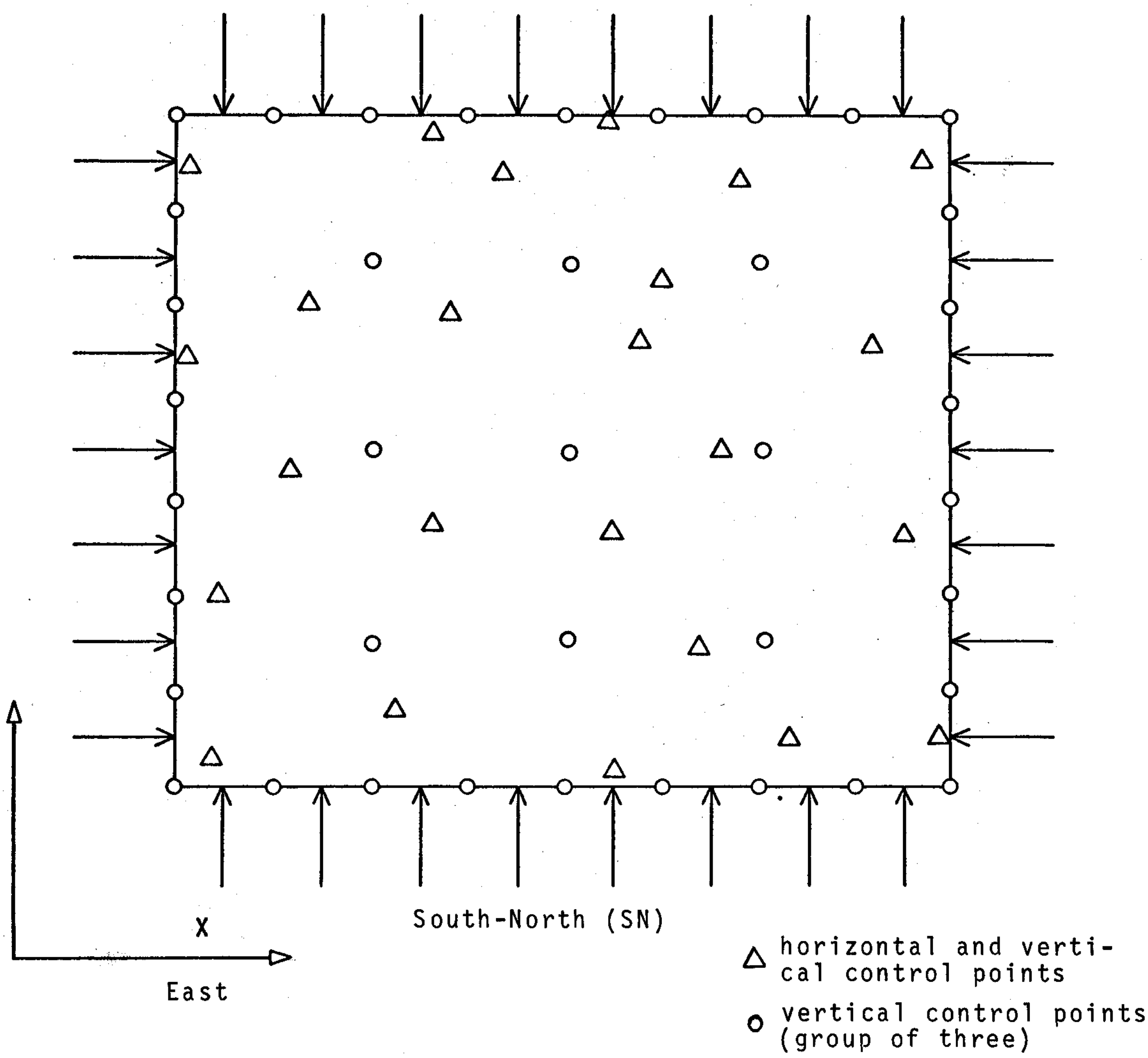
Parameters	Horizontal control *)	Overlap	σ_o - µm -	$\mu_{x,y}$ - µm -	μ_z	$\mu_{x,y}$	μ_z - cm -	$\varepsilon_{x,y\,max}$	$\varepsilon_{z\,max}$
None	dense perimeter control	1-fold	3.0	7.3	11.2	5.7	8.5	42.9	29.4
		2-fold	3.6	4.4	8.2	3.4	6.3	10.5	15.7
12 by blocks	dense perimeter control	2-fold	2.7	3.3	8.4	2.5	6.4	9.3	17.1
		4-fold	2.8	3.0	7.9	2.3	6.1	7.2	15.4
Selection by blocks	dense perimeter control	1-fold	2.4	4.9	10.1	3.8	7.8	18.6	23.7
		2-fold	2.7	3.3	8.3	2.6	6.4	9.3	17.0
12 by strips	dense perimeter control	1-fold	2.2	4.7	9.8	3.6	7.5	12.8	21.9
Selection by strips	dense perimeter control	1-fold	2.3	4.4	9.4	3.4	7.2	13.8	21.2
		2-fold	2.6	2.6	8.1	2.0	6.2	7.0	18.0
Selection by strips	trig. points	1-fold	2.3	4.4	9.4	3.4	7.2	12.3	21.4
		2-fold	2.6	2.4	7.7	1.8	5.9	5.0	13.6

*) Vertical control: 24 trigonometric points + 38 additional groups of three.

ε = maximum coordinate error at check points

$\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$, μ_x, μ_y = rms residual errors at check points

Figure 1 APPENWEIER, GENERAL OUTLINE OF FLIGHTS AND GIVEN CONTROL POINTS



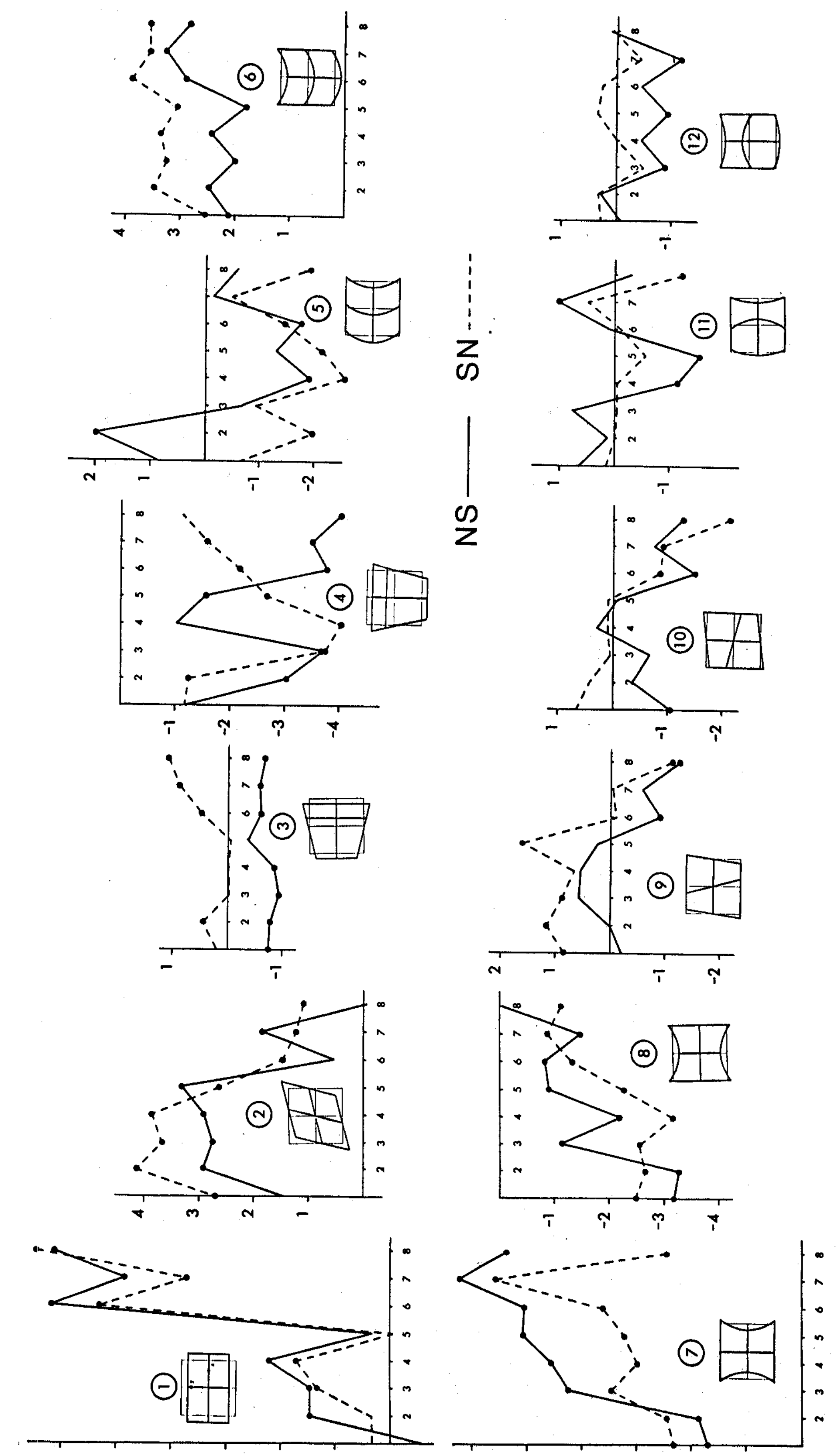
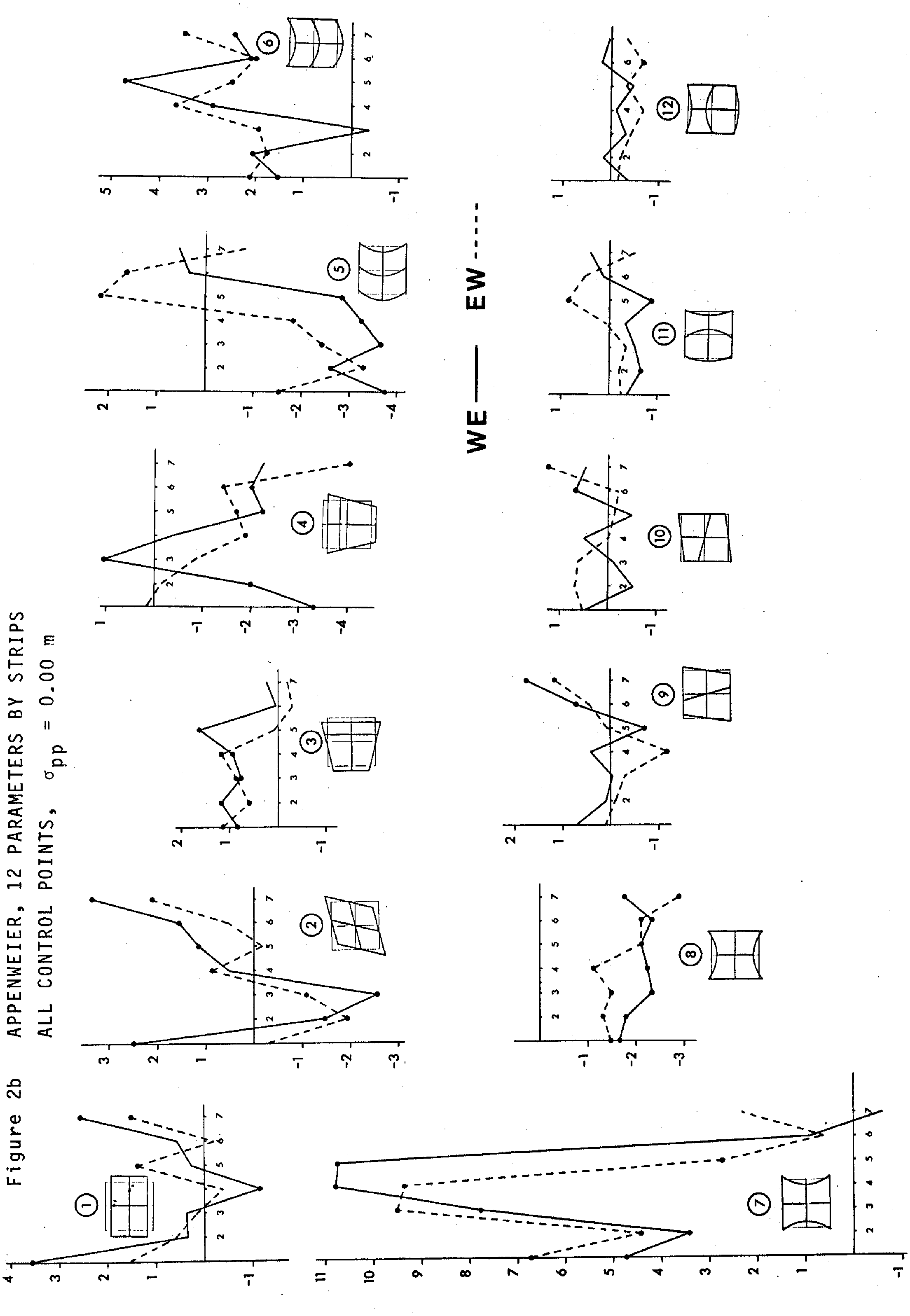


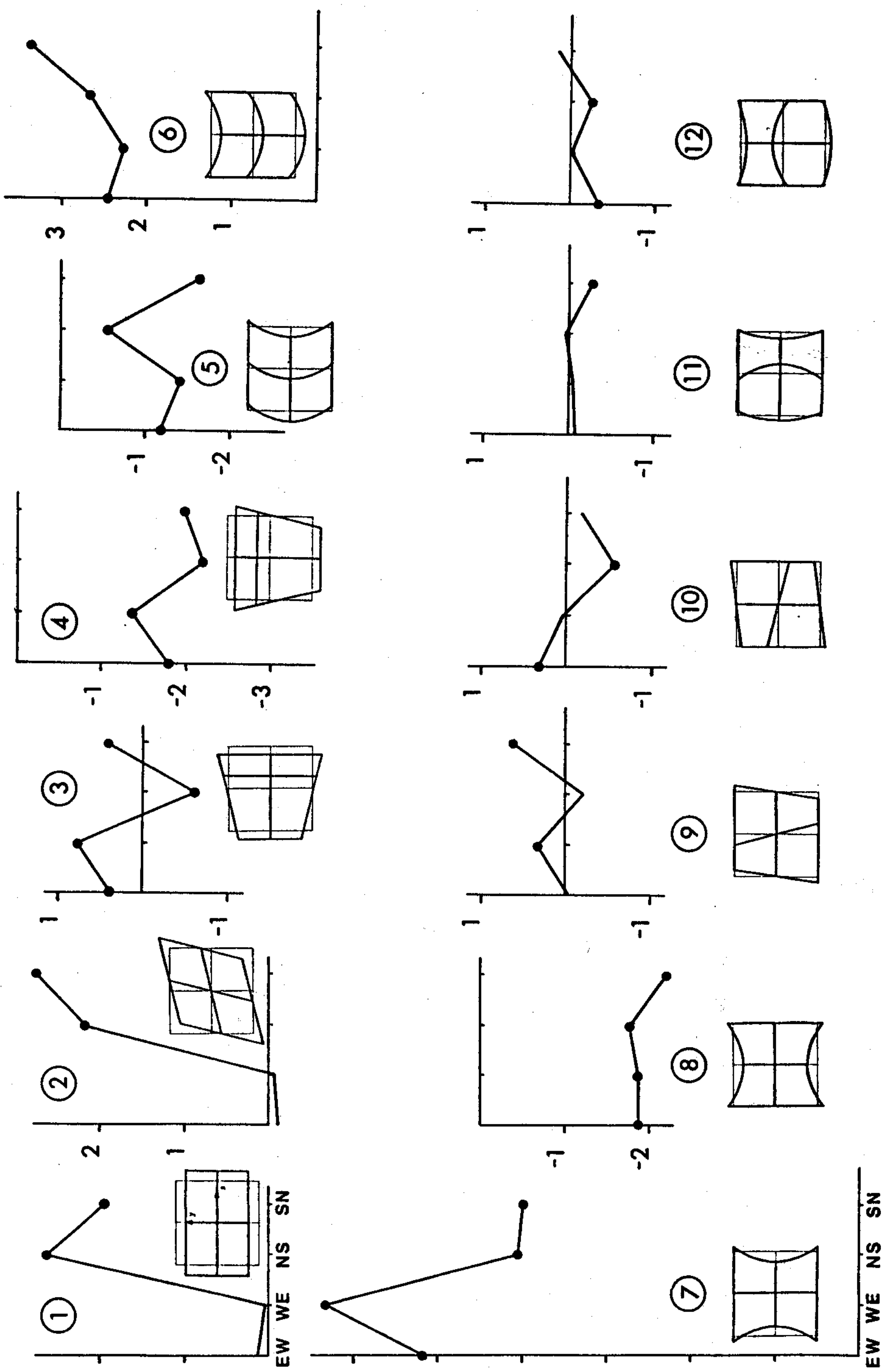
Figure 2a APPENWEIER, 12 PARAMETERS BY STRIPS
ALL CONTROL POINTS, σ_{pp} = 0.00 m

Figure 2b APPENWEIER, 12 PARAMETERS BY STRIPS
ALL CONTROL POINTS, σ_{pp} = 0.00 m



Photogrammmetrische Woche 1979
Institut für Photogrammetrie
Schriftenreihe, Heft 6, 1980

Figure 3 APPENWEIER, 12 BLOCK INVARIANT PARAMETERS
DENSE PERIMETER CONTROL, σ_{pp} = 0.01 m



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Abstract

After some initial remarks on the theory and the programming of additional parameters in bundle block adjustments, the paper presents results from the Appenweier test project. These are discussed, above all, with regard to selection, effectiveness and constancy of the parameters. In addition, the problems of determinability and statistical significance are discussed. An accuracy of image coordinates of 2 µm confirms the effectiveness of the technique.

Neue Ergebnisse der Bündelblockausgleichung mit zusätzlichen Parametern

Zusammenfassung

Nach einigen Vorbemerkungen zur Theorie und zur Programmierung zusätzlicher Parameter in der Bündelblockausgleichung werden Untersuchungen mit dem Testblock Appenweier vorgestellt. Insbesondere werden Auswahl, Wirksamkeit und Konstanz der Parameter analysiert und die Überprüfung auf Bestimmbarkeit und Signifikanz erläutert. Die erreichte Bildkoordinatengenauigkeit von 2 µm bestätigt die Wirksamkeit der Methode.

Nouveaux résultats de la compensation par faisceaux perspectifs avec des paramètres supplémentaires

Résumé

Après quelques remarques préalables sur la théorie et la programmation de paramètres additionnels dans la compensation par faisceaux perspectifs, nous exposons les études qui ont été faite avec le bloc test "Appenweier". Nous y analysons en particulier le choix des paramètres, leur utilité et leur constance et nous expliquons comment nous vérifions leur déterminabilité et leur signification. La précision de 2 µm obtenue sur les coordonnées-image confirme l'efficacité de la méthode.

Nuevos resultados de la compensación de un bloque de haces con parámetros adicionales

Resumen

Después de algunas observaciones preliminares sobre la teoría y la programación de parámetros adicionales en la compensación de bloques de haces se presentan las investigaciones efectuadas con el bloque de prueba Appenweier. Ante todo, se analizan la elección, eficacia y constancia de los parámetros y se explican su comprobación de la posibilidad de determinación y la significancia. La exactitud alcanzada de las coordenadas de la imagen de 2 µm confirma la eficiencia del método.

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