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THE EFFECT OF ANGULAR FIELD ON HORIZONTAL AND VERTICAL ACCURACY IN PHOTOGRAMMETRIC PLOTTING

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1. Introduction

A very important part for the planning of a photogrammetric project is the question of the type of aerial camera to be used. The answer is not obviously simple because it depends on a number of details which must be considered. Besides others one has to mention

a) the application of the photographs, i.e. if they are used for topographical purposes or for aerial triangulation,

b) the area to be covered,

c) the photo scale,

d) the flying height limit,

e) the nature of the object and

f) last not least the desired accuracy.

Considering the economic aspects to be the most important ones in most cases that camera should be used which can cover the largest area by one photo. This, of course, would be the camera with the largest aperture angle.

The camera manufacturers realized this already very early and started to develop cameras with a smaller focal length. At the beginning, there were a lot of difficulties because of the occurrence of a large lens distortion and of a relatively poor image quality. Finally in 1968 at the International Congress for Photogrammetry at Lausanne the companies ZEISS, WILD and JENOPTIK presented practically at the same time new super-wide angle cameras with small distortion values and an acceptable image quality. Nevertheless, this type of camera up to now is applied mainly for small scale mapping while it is rarely used for large scale applications.

The main reason for this could be the fact that the accuracy characteristics of the different camera types are not yet known well enough. Although there exist some investigations, especially by the camera manufacturers, they are not sufficiently approved by empirical tests. In this respect LÖSCHER |3|, MEIER |4| and WÜRTZ |7| have to be mentioned who in 1963/64 investigated the problem of the so-called optimum aperture angle. MEIER |5| has recently extended his work and also performed some test flights. To a certain degree, his results may be regarded as a basis for the following investigations.

This paper tries to show up empirically the relation between the photogrammetric accuracy and the angular field of aerial cameras. For this purpose a number of photos of a test flight taken with different types of cameras have been used. In the first part the theoretical accuracy of the photogrammetric model coordinates with respect to the aperture angle of the camera will be derived. The following main part deals with the practical results of the test where also some comments are made on the amount of systematic errors. In the last part some conclusions will be discussed which could be drawn from the results.

2. Theoretical accuracy of photogrammetric model coordinates

In the standard case of stereo-photogrammetry we get the following relations between the model coordinates and the measured image coordinates. The x-coordinate is pointing in flight direction and b means the base length.

$$X = b \cdot \frac{x'}{x' - x''} \tag{1}$$

$$Y = 0.5 \cdot b \cdot \frac{y' + y''}{x' - x''} \tag{2}$$

$$Z = b \cdot \frac{c}{x' - x''} \tag{3}$$

Assuming the orientation parameters to be errorfree the application of the law of error propagation on the formulae (1) to (3) yields the variances of the model coordinates (referring to the image scale):

$$\sigma_X^2 = \frac{1}{b^2} \{ (b-x')^2 \sigma_{x'}^2 + x'^2 \sigma_{x''}^2 \} \tag{4}$$

$$\sigma_Y^2 = \frac{1}{4b^2} \{ b^2 \sigma_{y'}^2 + b^2 \sigma_{y''}^2 + \frac{(y'+y'')^2}{b^2} \sigma_{x'}^2 + \frac{(y'+y'')^2}{b^2} \sigma_{x''}^2 \} \tag{5}$$

$$\sigma_Z^2 = \frac{1}{b^2} (c^2 \sigma_{x'}^2 + c^2 \sigma_{x''}^2) \tag{6}$$

with $b = x' - x''$ = model base

With the simplifications

$$x' = x \tag{7}$$

$$y' = y'' = y \tag{8}$$

$$\sigma_{x'} = \sigma_{y'} = \sigma_{x''} = \sigma_{y''} = \sigma_k \tag{9}$$

the final formulation is obtained

$$\sigma_X^2 = \frac{1}{b^2} \{ (b-x)^2 + x^2 \sigma_k^2 \} \tag{10}$$

$$\sigma_Y^2 = \frac{1}{b^2} \{ \frac{b^2}{2} + 2y^2 \sigma_k^2 \} \tag{11}$$

$$\sigma_Z^2 = \frac{1}{b^2} \cdot 2c^2 \cdot \sigma_k^2 \tag{12}$$

Hence it follows that the standard deviation of the horizontal model coordinates x and y in fact is a function of the image coordinates but it is not depending on the aperture angle. On the other hand the standard deviation of the height is inversely proportional to the base-height relation.

By integrating the formulae (10) and (11) over the entire model area the following standard deviations of the model coordinates for 60 % longitudinal overlap and a photo size of 23 x 23 cm^2 will be received:

$$\sigma_X = 0.82 \cdot \sigma_k \tag{13}$$

$$\sigma_Y = 1.08 \cdot \sigma_k \tag{14}$$

$$\sigma_Z = 0.0154 \cdot c_{mm} \cdot \sigma_k \tag{15}$$

Assuming a proper standard deviation of the image coordinates of σ_k = 5 µm an accuracy behaviour will be achieved as shown in figure 1. The amount of the standard deviation of the y-coordinate exceeds that one of the x-coordinates by about 30 %. The relation of the standard deviation in height for the applied cameras with a focal length of 30 cm, 21 cm, 15 cm, and 8.5 cm (which refers to an aperture angle of 55^0, 75^0, 95^0 and 125^0) is about 3.6 : 2.5 : 1.8 : 1.0. That means, the wider the aperture angle is the more accurate the height will be.

But the above derivation is only valid for the assumed conditions (standard case, error-free orientation parameters, only irregular image coordinate errors). In practice it is therefore possible that more or less different results will be obtained. In order to consider the practical relations as far as possible MEIER |4|, |5| tries to find a formulation which includes also the physical influences of the photographical process.

3. The mathematical accuracy model by MEIER

MEIER estimates three error components by which the image coordinates are influenced, i.e. one for film shrinkage, one for the roughness of the emulsion and of the emulsion carrier, and one for the optical errors. The result of his study is a mathematical model which shows up the relation between the coordinate accuracy and the angular field of the camera, the image size, and the overlapping.

In a next step the mathematical model is checked and improved by a practical test. A test field was photographed with four different types of cameras, resulting in two models for each camera. The models were measured at a PLANIMAT instrument and were corrected by a subsequent affine transformation. The standard deviations of the coordinates were found by comparing the transformed model coordinates with their given values.

Depending on this empirical accuracy MEIER changes his mathematical model in such a way that it corresponds as sufficiently as possible with the test results. The new mathematical model 2.1 is shown in figure 2.

In contrary to the theoretical accuracy of figure 1 the standard deviations of the coordinates x and y are no longer independent from the image angular field but are increasing with an increasing aperture angle. In addition, the difference between σ_x and σ_y is not constant but is increasing from about 50 % up to about 100 %. The dependency of the accuracy of the z-coordinates from the aperture angle is not as serious as before. The standard deviation in height of the 30 cm - and of the 8.5 cm - camera is now in the ratio of about 2 : 1.

In MEIER's opinion the mathematical model describes the practical results sufficiently although there are some significant differences. The relationship between the accuracy of the model coordinates and the aperture angle were therefore discovered.

But just for the mentioned significant differences additional investigations are required. In the following the results of a new test will be presented with which additional knowledge should be obtained.

4. Results of practical tests

4.1 Photo flight and restitution procedure

For the new test the test field Rheidt of the University of Bonn was used. This test field has been established especially for investigations related to the photogrammetric image or model. It consists of 41 groups of points with 3 points each, whose coordinates show standard deviations between 6 mm and 12 mm in planimetry and of less than 10 mm in height. The points are permanently marked and their coordinates are determined every year again. For more details see KUPFER |2|.

In spring 1974 the photo flight was performed using 3 different types of cameras with 85 mm, 208 mm, and 305 mm focal length. With each camera 8 models were flown with a photo scale of 1:11 000. It didn't seem necessary to employ also a wide-angle camera because there existed already 47 models from the same

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test field being flown in 1969. Table 1 contains the data of all the photo flights which were used for the present investigation.

The measurement was done at a ZEISS PSK2 stereo-comparator using glass dia-positives. The photos which MEIER already used in 1969 for his test and which were measured at a PLANIMAT were again observed at the PSK. In this case the original negatives were measured.

The ground coordinates of all model points were computed by a numerical ab-solute orientation based on the bundle method in which all measured points were introduced as control points. The image coordinates were corrected for the influence of lens distortion, of affine film shrinkage, and of earth curvature and refraction. In addition, all models of the 1974 photo flight and the 47 models from 1969 were computed by means of a two-step method with separate relative and absolute orientation. In this case for computational reasons only 13 symmetrically distributed control points have been introduced.

For checking the accuracy the mean square values of the residuals between the adjusted coordinates and their known values were used. Finally the average of the mean square values of all models taken with the same camera allows to compare the accuracy of different types of cameras.

4.2 Results of the 1969 photo flight

As mentioned above the photos of the 1969 flight first were measured as at PLANIMAT. The results are shown in figure 3 in graphical form. This figure also contains the confidence intervals based on a χ^2 distribution for a 5 % level of significance.

Compared with the diagrams of figure 1 and 2 especially the favourable height accuracy of the 21 cm camera and the favourable planimetric accuracy of the 8.5 cm camera have to be pointed out. In addition, the standard deviation in height as before increases with decreasing aperture angle and the standard deviation in planimetry decreases with an increasing aperture angle. But these results are not only influenced by the errors of the camera but also of the analogue instrument which depend on the angular field, too. Therefore the photos have been remeasured at a PSK stereo-comparator, and model coordinates have been computed by using the bundle orientation method.

Figure 4 shows the result in the usual form. In table 2 the results of the PLANIMAT- and of the PSK2-measurements are compared with each other. As ex-pected, the results of the stereo-comparator are more accurate in all three coordinate directions - with only two exceptions. The standard deviation of the height now decreases continously with an increasing aperture angle. On the other hand, the behaviour of the planimetric accuracy is still not very constant. The main reason for this rather insufficient result could be the fact, that from each camera only 2 models have been investigated. This is a very small sample which may not be satisfactory for a significant statement. Therefore all further conclusions shall be drawn from the results of the 1974 photos only.

4.3 Results of the 1974 photo flight

Two methods were used for the numerical orientation of the comparator obser-vations, i.e. the bundle method which theoretically should be the most rigorous orientation procedure, and the orientation by steps with separate relative and absolute orientation.

Table 3 contains the results of both methods. In addition, the results are shown in the graph of figure 5 and 6. Considering first the bundle method, one can realize that the planimetric accuracy is nearly independent of the aper-ture angle and the y-coordinate is on an average by the factor 1.25 less accurate than the x-coordinate. The standard deviations of the z-coordinates decrease continously between the aperture angles 56° and 125° by the relation-ship 2.5 : 1.6 : 1.3 : 1.0.

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The results of the wide angle camera fit very well into the curve of the accuracy although they originate in a quite different photo flight from 1969. But the standard deviations are based on 47 models so that their statistical significance is very good which can also be seen at the very narrow confidence interval. From this the conclusion could be drawn that it is really better to choose the sample as large as possible than to look for most homogeneous conditions and keep the sample small.

The 1974 results correspond quite well with the theoretical accuracy shown in figure 1, especially when planimetry is concerned. The difference between the standard deviations of the x- and y-coordinates of about 25 % is nearly the same as the theoretical one which is about 30 %. On the contrary, the accuracy in height is not increasing in the same way with increasing aperture angle as in the theoretical case where the ratio between normal angle and super-wide angle camera has been about 3.6 : 1.0.

When comparing the bundle method with the step-by-step method (figure 5 and 6) it is surprising that the planimetric coordinates of the step method are less accurate but that the heights are more accurate. The reason for this may be a different definition of the location of the adjusted points. The mean difference of the standard deviations of the x- and y-coordinates is practically the same for both methods. But the curve of the height accuracy of the bundle method is flatter, and its ratio of the standard deviations of the z-coordinate for the four types of cameras is about 2.2 : 1.3 : 1.2 : 1.0.

There are also very large differences between the empirical results and MEIER's mathematical model 2.1. In fact, the flat curves of the height accuracy are very similar in both cases but the numerical values of the mathematical model are about twice as large. In addition, the empirical results do not show a constant increase of the planimetric accuracy with increasing aperture angle.

Now, the practical test results are of course influenced by systematic errors which may explain to some extent the relatively bad correspondence between theory and practice. In the following the systematic components which may be contained in the results shall be further investigated. Before this it is necessary to introduce and explain some terms which will be used.

4.4 Investigation of systematic errors

In general, a statistical experiment contains three components which are called the "deterministic", the "correlated" and the "random" component. In conventional terms the deterministic and the correlated component are not properly distinguished but are combined in the term of the systematic error. Anyway, in this conventional sense one could speak of a "strictly systematic" and a "locally systematic" error. Some other terms which are also often used are "trend" for the deterministic component, and "observational error" or "noise" for the random component.

Since it is practically impossible to determine all three components at once, first of all the trend will be eliminated and then the random component will be determined.

4.4.1 Determination of the trend

Considering the given practical test the trend may be defined as that systematic error which shows the same amount and sign in all modells of one camera type (that means which is constant when respecting the complete experiment). It is then relatively simple to calculate the trend. All the signalized points of the test field Rheidt form a regular rectangular grid. For the test the photo scale has been chosen in such a way that the test field was covered by two models each. If the photos were carefully taken each model contains exactly the same number of points with the same distribution over the model. This is independent of the flight direction and of the actual area of the test field which is covered by the model. That means, each model should contain 23 groups of points with 3 points each. Figure 7 shows these 23 point groups and their distribution over the model area.

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Considering now that all models are put one on top of the other one can calculate the sum of the residuals of all those points which are located in the same position within the model. If now this sum is not equal to zero this indicates the presence of a trend and its value is equal to the amount of the trend in that particular point of a model. This method is described in detail in STARK |6|. All the results of the 1974 photo flight have been analysed for the existence of a deterministic component. For the bundle adjustment the result of all 4 types of cameras is shown in figure 9 to 12 in graphical form. The diagrams also contain the mean square values of the trend of all 23 point groups of a model.

For each camera type a significant systematic error is obtained. The amount of the trend in planimetry is about 2 µm to 3 µm and in height it is about 4.5 µm, for the normal angle camera even 9 µm. The main planimetric model deformation seems to be in form of a trapezoid while in height we get a bend in y-direction. Only the 30 cm camera shows some kind of a confusing deformation which especially in height is not very plausible. May be that the sample of 8 models is still too small for a significant analysis.

The orientation by steps shows the same type of deformation. It's amount in planimetry is only a little bit larger and in height a little bit smaller. That means, that the differences between the two orientation methods are caused by systematic errors only.

4.4.2 Determination of the random component

The calculation of the random component can be performed by means of the covariance function which, for instance, is used with the application of the least-squares interpolation in photogrammetry (see KRAUS / MIKHAIL |1|). The vertex of this covariance function represents the covariance of pairs of points whose distance is infinitely small, that means, it is equivalent to the systematic error. For the given data this systematic error contains the deterministic component as well as the correlated component so it may be allowed to speak simply of the systematic component.

The covariance function is determined by all the covariances between two points each within a certain distance interval. As an interpolating curve between the covariances of different intervals a Gaussian function may be chosen (see figure 8). Anyway, the shape of the function is not important because in our case only the term C(0) which represents the vertex is required.

Since the systematic and the random component are independent variables the variance σ_r^2 of the random component may be computed as the difference of the variance V and the systematic component C(0)

$$\sigma_r^2 = V - C(0) \qquad (16)$$

The determination of the random component has been performed for both photo flights 1969 and 1974 and for both orientation methods. The result of the bundle method is represented for both photo flights in table 4. On the other hand, table 3 shows the comparison of the two orientation methods for the 1974 photo flight.

The random component σ_r for each type of camera is practically constant in all three cases. This meets the expectations because σ_r should be free of all systematic errors and therefore must be independent of the influences of the photo flight and the photographic process and of the orientation method. But at the same time it is proved that this method for the determination of the random component is properly working. Therefore, in the following only the results of the bundle orientation method of the 1974 photographs will be considered.

In planimetry the random component is about 2 µm to 3 µm and the difference between the x- and y-coordinate remains about 25 % as before. Also in height the continuously increasing tendency of the accuracy with increasing angular field is still existing. The ratio of the standard deviations in height between the four camera types changes only very slightly and is now for the random component about 2.3 : 1.7 : 1.5 : 1.0 (compared with 2.5 : 1.6 : 1.3 : 1.0 for the total error).

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Figure 13 shows the effect of the aperture angle on the random component. This diagram also contains the theoretically expected accuracy. In this case the standard deviation σ_k has been calculated from formulae (13) and (14) by

$$\sigma_k = \frac{\sigma_x}{0.82} \qquad (13a)$$

$$\sigma_k = \frac{\sigma_y}{1.08} \qquad (14a)$$

Both values of σ_k were corresponding very well. The mean value for all types of cameras was $\sigma_k = 2.5$ µm. Based on this value the theoretical expectations for σ_x, σ_y and σ_z have been evaluated from formulae (13) to (15). This case is marked in figure 13 with little lines.

Comparing the empirical and the theoretical accuracy a proper coincidence can only be seen for the planimetric coordinates. The curve of the empirical height accuracy is still much flatter than for the theoretical derivation. This means, that the obtainable accuracy in height by a super-wide angle camera with 8.5 cm principal distance is not as good as could be expected from theory.

The comparison with the mathematical model 2.1 by MEIER can only be relative because the numerical values are too different. However, both diagrams show that the standard deviations of the planimetric coordinates are increasing with an increasing aperture angle. But the empirical amount of about 35 % is by far smaller than that of the mathematical model with about 65 % in x- and 110 % in y-direction. The accuracy relation in height between the super-wide angle and the normal angle camera is also smaller for the mathematical model (1.8 : 1.0 compared with 2.3 : 1.0).

Therefore the mathematical model 2.1 does not seem to describe the accuracy relations of different types of aerial cameras in a satisfactory way. Anyway, it might be possible to find another mathematical formulation which fits better to the empirical results. But this was not the task of the present paper.

5. Conclusion

Although some significant discrepancies were found between theoretical expectations and empirical results it is possible to draw some conclusions. They are mainly based on the random component of the variances of the coordinates as they are shown in figure 12. These values proved to be almost constant for two different independent tests (1974 and 1969) and for different orientation methods. Therefore they allow to make some relatively definite statements.

As to the accuracy in height, it can be stated that the standard deviation of the z-coordinate is constantly decreasing with an increasing aperture angle of the camera. The accuracy relation between a normal angle camera (c = 30 cm) and a super-wide angle camera (c = 8.5 cm) is in the order of about 1 : 2. In addition, it seems to be certain that the planimetric accuracy is almost independent from the angular field of the camera. But the standard deviation of the x-coordinate is significantly smaller than that of the y-coordinate. The main difference is about 25 %.

If the location in space of a point is considered the super-wide angle camera yields the best result. It's standard deviation $\sigma_P = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_y^2}$ is only about one half of the normal angle camera. Nevertheless, when using a super-wide angle camera some concessions must be made on the image quality which especially at the edges of the image is not reaching the quality of cameras with smaller aperture angles. If best planimetric accuracy is required the normal angle camera may have some advantages. It's standard deviation $\sigma_L = \sqrt{\sigma_x^2 + \sigma_y^2}$ is at about 35 % smaller than that of the super-wide angle camera.

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These statements are no longer true when also the systematic errors are considered. Then in planimetry the normal angle camera is no longer superior. After the orientation by the bundle method it's standard deviation σ_L is even the greatest one of all cameras. But the height accuracy relations in principle remain the same.

The investigation on systematic errors showed up noticable terms. Even the amount of the deterministic component of about 2.5 μm in planimetry and 5 μm to 10 μm in height is not negligible. In order to make significant predictions it is absolutely necessary to consider or eliminate the systematic errors as far as possible.

The best way to do this is in combination with numerical orientation methods. Here the so-called self-calibrating adjustment procedures or the separate test field calibrations have to be mentioned. The investigations and tests which have been performed up to now seem to be of great promise. Other possibilities - which of course may hardly be realizable - could be tried with the camera production or at least with the laboratory calibration.

Finally it may be mentioned that of course it would be important to check or to confirm the above results by additional investigations. This applies especially to the field of aerial triangulation where the only comparable test, namly Oberschwaben, didn't yield satisfactory results when comparing the accuracy of wide angle block adjustments with super-wide angle block adjustments.

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Table 1: Flight specifications of the test field Rheidt

Type of camera	RMK 30/23	RMK 21/23	RMK 15/23	RMK 8,5/23	RMK 30/23	RMK 21/23	RMK 15/23	RMK 8,5/23
Focal length	305.35 mm	208.29 mm	153.20 mm	85.41 mm	305.24 mm	207.93 mm	153.20 mm	85.30 mm
Date of flight	5.4.1969				5.6.1974		12.6.1969	5.6.1974
Flying height above ground	3050 m	2100 m	1530 m	850 m	3350 m	2250 m	1650 m	940 m
Image scale	1 : 10 000				1 : 11000		1 : 10500	1 : 11000
Airplane	Aero-Commander				Aero-Commander			
Camera no.	110 403	20 208	111 680	111 164	118 411	115 739	111 680	20 281
Lens no.	Topar A 110 414	Toparon A 98 308	Pleogon A2 112 647	S-Pleogon A 112 333	Topar A 118 390	Toparon A 116 229	Pleogon A2 112 647	S-Pleogon A 120 326
Film	Aviphot Pan 30 PE				Aviphot Pan 30		Kodak-Plus-X	Aviphot Pan 30
Overlap	90 %				90 %		60 %	90 %

Table 2: Comparison of the accuracy between PLANIMAT- and PSK2-measurements

Root mean square values of 2 models each
Image scale 1:10 000

	PLANIMAT				PSK 2			
Focal length mm	σ_x µm	σ_y µm	σ_z µm	σ_z ‰ v.h	σ_x µm	σ_y µm	σ_z µm	σ_z ‰ of h
305	4.8	5.0	21.1	0.069	6.7	5.2	16.6	0.054
210	5.8	7.8	11.0	0.052	5.0	6.3	12.5	0.060
153	5.8	7.4	11.3	0.074	3.4	4.1	8.2	0.054
85	4.6	6.7	12.0	0.141	4.2	4.9	6.3	0.074

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Table 3: Comparison of the accuracy between the bundle and the step-by-step method

1974 Photo flight, Image scale 1:11 000
Root mean square values of 8 models each

	Focal length mm	Standard deviation σ_x µm	σ_y µm	σ_z µm	Random component σ_{r_x} µm	σ_{r_y} µm	σ_{r_z} µm	Systematic component σ_{s_x} µm	σ_{s_y} µm	σ_{s_z} µm
	Bundle method (all points used as control points)									
	305	3.9	4.7	14.4	1.7	2.3	7.1	3.5	4.0	12.5
	210	3.5	4.5	9.3	1.9	2.5	5.3	3.0	3.8	7.7
1)	153	3.6	4.2	7.4	2.4	2.8	4.5	2.7	3.1	5.9
	85	3.6	4.8	5.8	2.3	3.1	3.1	2.7	3.7	5.0
	Step-by-step method (13 regularly distributed control points)									
	305	4.7	6.0	12.7	1.9	2.4	7.0	4.3	5.5	10.6
	210	5.1	6.3	7.8	1.9	2.4	5.2	4.7	5.9	5.8
1)	153	4.6	5.3	6.9	2.5	2.8	4.4	3.9	4.6	5.2
	85	4.8	6.3	5.8	2.5	3.2	2.8	4.1	5.4	5.1

1) 1969 Photo flight, Image scale 1:10 500
Root mean square values of 47 models each

Table 4: Accuracy of photogrammetric coordinates as a function of the aperture angle
Results of numeric orientations of models by the bundle method

	Focal length mm	Aperture angle β°	Number of check points	σ_0 µm	Standard deviation σ_x µm	σ_y µm	σ_z µm	Random component σ_{r_x} µm	σ_{r_y} µm	σ_{r_z} µm	Systematic component σ_{s_x} µm	σ_{s_y} µm	σ_{s_z} µm	Confidence interval α = 5 % lower limit in %	upper limit in %
	1969 Photo flight, Root mean square values of 2 models each (image scale 1:10 000)														
	305	56	121	5.2	6.7	5.2	16.6	2.3	2.3	7.9	6.3	4.7	14.6	11	15
	210	75	150	4.8	5.0	6.3	12.5	2.1	3.4	6.2	4.5	5.4	10.9	10	13
	153	94	137	3.7	3.4	4.1	8.2	1.9	2.6	4.5	2.7	3.1	6.9	10	14
	85	125	114	4.7	4.2	4.9	6.3	2.0	3.2	3.5	3.7	3.7	5.2	11	15
	1974 Photo flight, Root mean square values of 8 models each (image scale 1:11 000)														
	305	56	481	4.2	3.9	4.7	14.4	1.7	2.3	7.1	3.5	4.0	12.5	6	7
	210	75	484	4.0	3.5	4.5	9.3	1.9	2.5	5.3	3.0	3.8	7.7	6	7
1)	153	94	3093	3.7	3.6	4.2	7.4	2.4	2.8	4.5	2.7	3.1	5.9	2	3
	85	125	373	5.0	3.6	4.8	5.8	2.3	3.1	3.1	2.7	3.7	5.0	7	8

1) 1969 Photo flight, Root mean square values of 47 models (image scale 1:10 500)

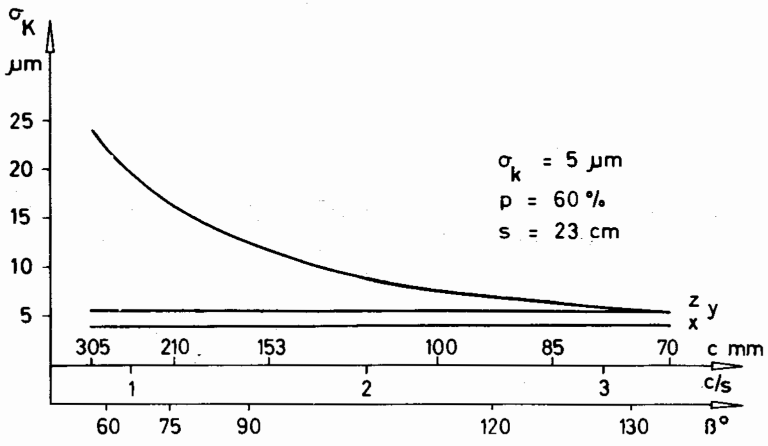


Figure 1: Theoretical accuracy of model coordinates

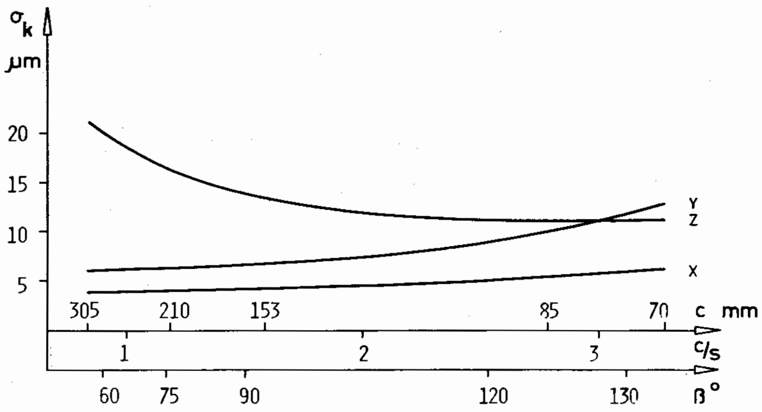


Figure 2: Mathematical model 2.1 by MEIER

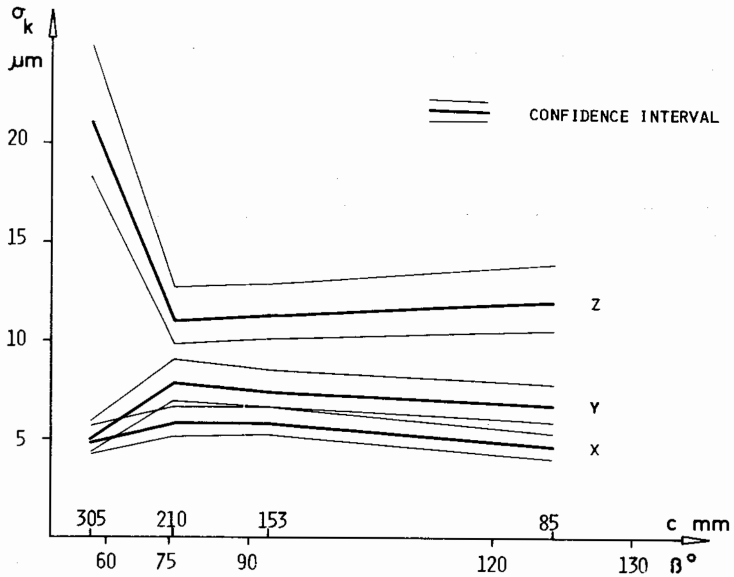


Figure 3: Standard deviation of the coordinates 1969 photo flight, PLANIMAT measurement

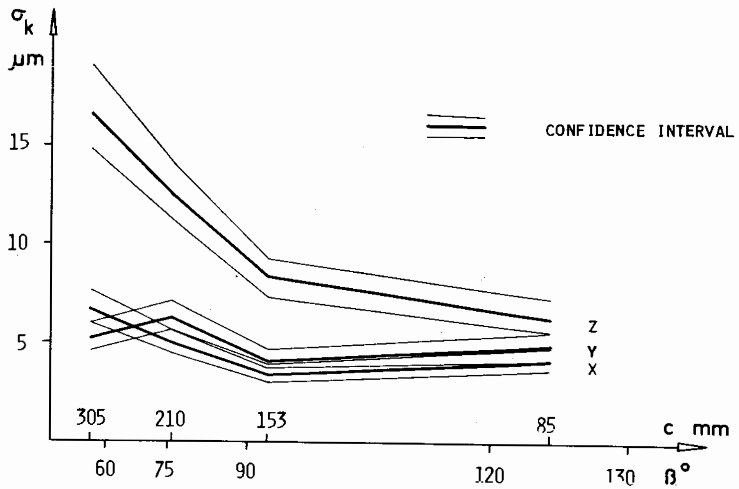


Figure 4: Standard deviation of the coordinates 1969 photo flight, PSK measurement

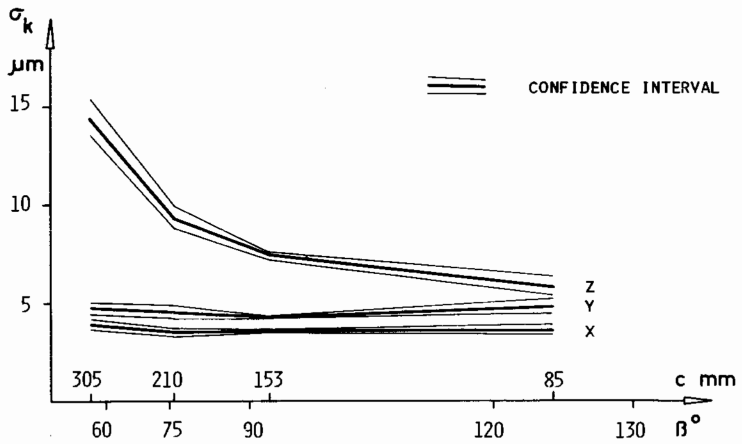


Figure 5: Standard deviation of the coordinates 1974 photo flight, bundle method

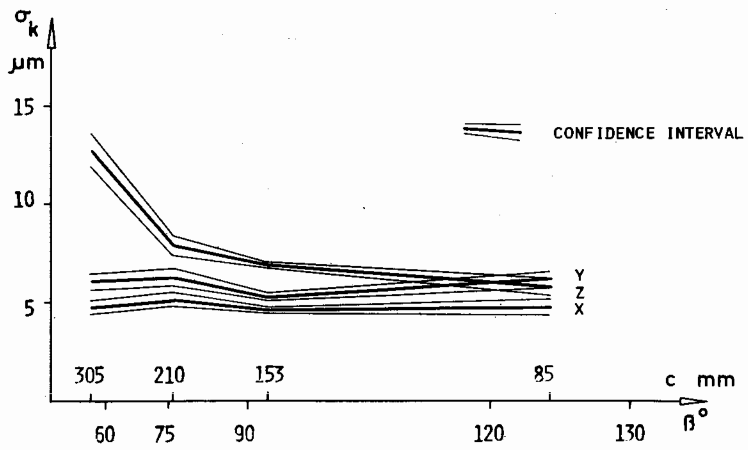


Figure 6: Standard deviation of the coordinates 1974 photo flight, step-by-step method

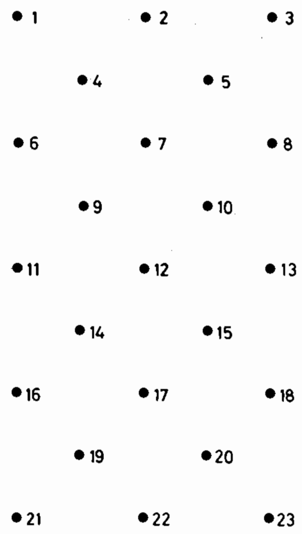


Figure 7: Arrangement of the 23 point groups

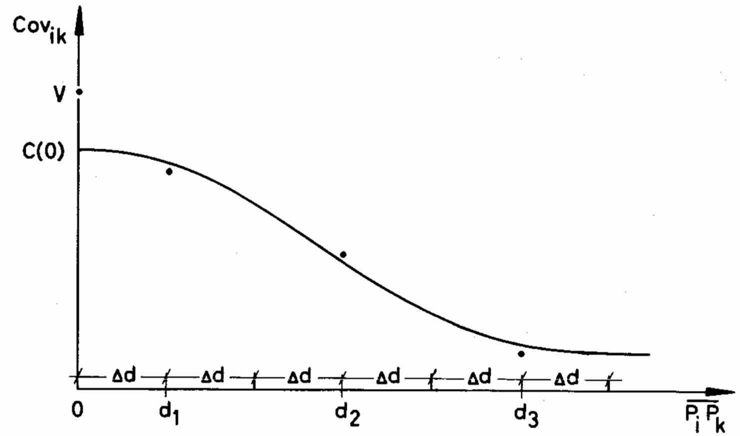
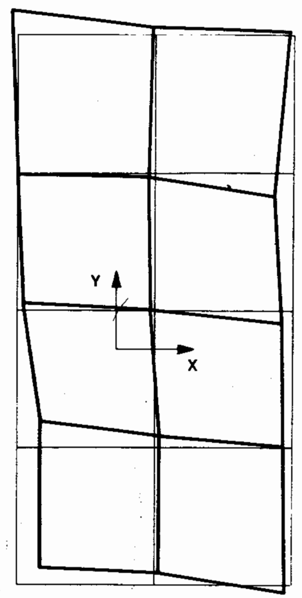


Figure 8: Covariance function



Root mean square values

t_x = 2.4 µm

t_y = 3.3 µm

t_z = 4.6 µm

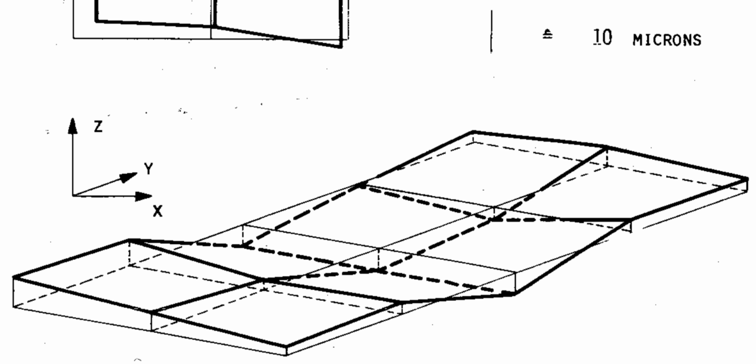
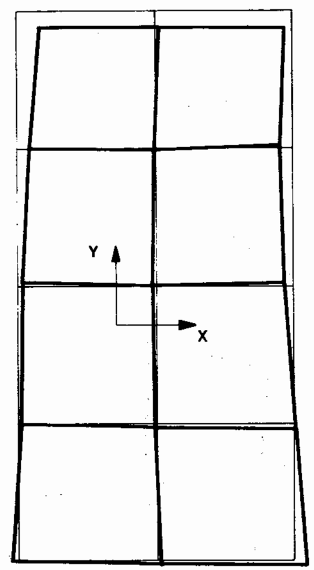


Figure 9: Trend for RMK 8.5/23
Orientation by the bundle method



Root mean square values

t_x = 2.0 µm

t_y = 1.8 µm

t_z = 4.3 µm

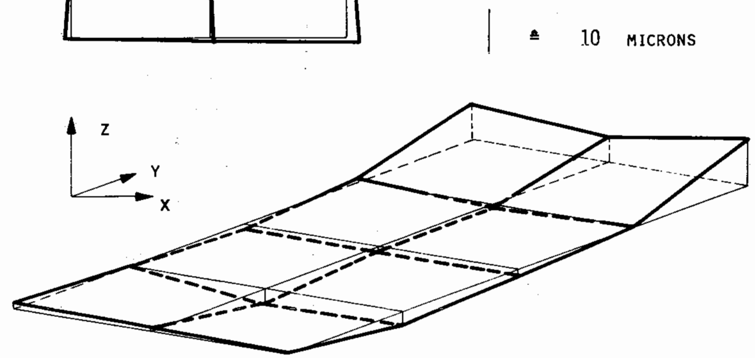
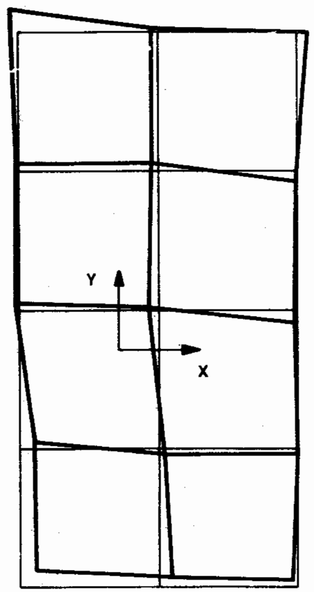


Figure 10: Trend for RMK 15/23
Orientation by the bundle method



Root mean square values

t_x = 2.0 µm

t_y = 2.5 µm

t_z = 4.3 µm

≙ 10 MICRONS

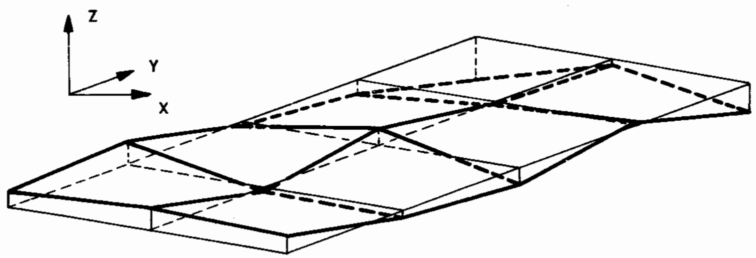
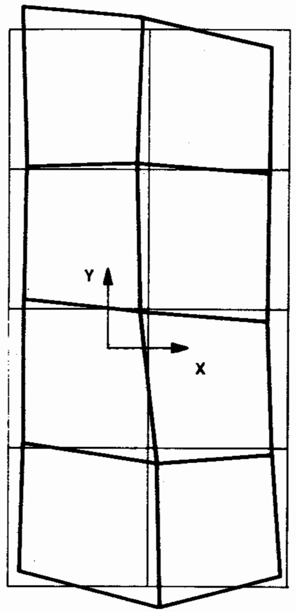


Figure 11: Trend for RMK 21/23
Orientation by the bundle method



Root mean square values

t_x = 3.0 µm

t_y = 2.8 µm

t_z = 8.9 µm

≙ 10 MICRONS

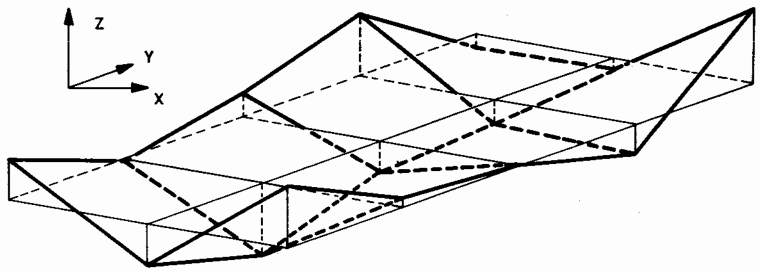


Figure 12: Trend for RMK 30/23
Orientation by the bundle method

Photogrammetrische Woche 1975
Institut für Photogrammetrie
Schriftenreihe, Heft 2, 1976

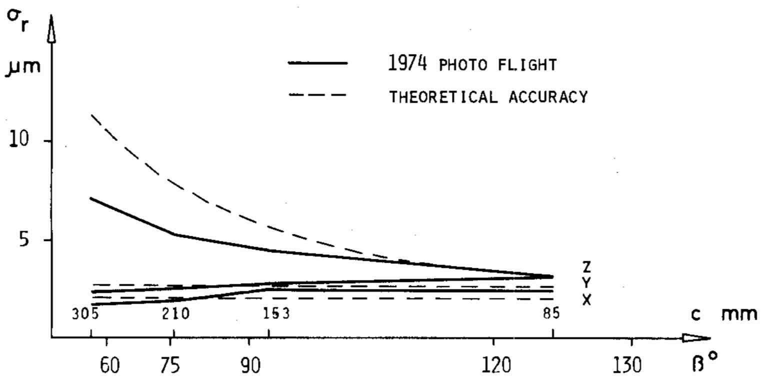


Figure 13: Random component
Orientation by the bundle method

Photogrammetrische Woche 1975
Institut für Photogrammetrie
Schriftenreihe, Heft 2, 1976

Zusammenfassung

Ausgehend von Testfeldbefliegungen mit vier verschiedenen Kammern und anschließender analytischer Einzelmodelleinpassung wird der Einfluß des Öffnungswinkels der Aufnahmekammer auf die Lage- und Höhengenauigkeit photogrammetrischer Auswertungen empirisch untersucht. Die Ergebnisse werden in Abhängigkeit des Bildwinkels dargestellt, diskutiert und mit bereits vorhandenen Ergebnissen verglichen. Weiterhin wird ein mathematisches Modell von Dr. MEIER überprüft, das die Genauigkeit als Funktion des Bildwinkels angibt. Aus den mittleren Lage- und Höhenfehlern wird schließlich der systematische Anteil herausgerechnet, und der unregelmäßige Anteil wird für die weiteren Betrachtungen herangezogen.

Grundsätzlich zeigt sich, daß der mittlere Höhenfehler erwartungsgemäß mit steigendem Bildwinkel stetig abnimmt, während die Lagegenauigkeit praktisch unabhängig vom Bildwinkel ist. Dabei ist der mittlere Koordinatenfehler in y-Richtung stets größer als derjenige in x-Richtung. Mit den Ergebnissen gelingt es allerdings nicht, ein plausibles mathematisches Modell abzuleiten.

Die Frage nach dem genauesten Luftbildkammertyp läßt sich nicht eindeutig beantworten. Zwar zeigt der Test, daß die Überweitiwnkelkammer den kleinsten mittleren Punktfehler (x,y,z) liefert, da sie in der Höhe eindeutig am genauesten ist und in der Lage nur unwesentlich ungenauer als die übrigen Kammertypen, doch stehen dieser Beurteilung noch eine etwas schlechtere Bildqualität und gegenteilige Ergebnisse aus der Aerotriangulation gegenüber. Bei höchsten Ansprüchen an die Lagegenauigkeit könnte der Normalwinkelkammer der Vorzug gegeben werden, da sie den geringsten unregelmäßigen Anteil des mittleren Koordinatenfehlers in x und y aufweist. Bei Berücksichtigung des systematischen Anteils ist jedoch die Normalwinkelkammer nicht mehr überlegen. Die Frage der Elimination systematischer Fehler erlangt daher in diesem Zusammenhang besondere Bedeutung.

Abstract

An empirical study of the effect of the aperture angle of aerial cameras on horizontal and vertical accuracy in photogrammetric plotting is made on the basis of test flights performed with four different cameras and subsequent analytical orientation of individual models. The results are shown as a function of angular field, discussed and compared with existing results. In addition, a mathematical model by Dr. MEIER is examined, which gives accuracy as a function of angular field. Finally, the systematic component is computed from the mean squarehorizontal and vertical errors, the irregular component being used as a basis for further considerations.

In principle, it is found that the mean square vertical error decreases with increasing angular field, as had been expected, while horizontal accuracy is practically independent of angular field, the mean square error in the y-coordinate being generally larger than in the x-coordinate. The results obtained are, however, insufficient for deriving a plausible mathematical model.

The question which type of aerial camera is the most accurate cannot be clearly answered. Although the test shows that ultra wide-angle cameras give the smallest point error (x,y,z) because their vertical accuracy is clearly superior while their horizontal accuracy is only slightly less than that of the other camera types, their image quality is still not entirely comparable to that of other cameras and they perform less satisfactorily in aerial triangulation work. For maximum horizontal accuracy requirements, preference might be given to normal-angle cameras since these have the smallest irregular component of the mean coordinate error in x and y. However, if allowance is also made for the systematic component, normal-angle cameras are no longer superior. The elimination of systematic errors is therefore particularly important in the context.

Photogrammetrische Woche 1975
Institut für Photogrammetrie
Schriftenreihe, Heft 2, 1976

Résumé

L'influence de l'angle de champ sur la précision planimétrique et altimétrique des restitutions photogrammétriques fait l'objet d'une étude empirique, basée sur des survols de test exécutés avec quatre chambres aérophotogrammétriques de types différents, ainsi que sur l'orientement analytique des modèles individuels. Les résultats sont présentés en fonction de l'angle de champ, puis discutés et comparés avec des données déjà disponibles. L'expose examine en outre un modèle mathématique du Dr. MEIER qui indique la précision en fonction de l'angle de champ. La composante systématique est calculée finalement à partir des erreurs planimétriques et altimétriques moyennes, tandis que la composante irrégulière sert à l'énoncé d'autres conclusions.

Comme on le supposait, l'erreur altimétrique moyenne décroît en principe au fur et à mesure que l'angle de champ augmente. La précision planimétrique reste par contre indépendante de l'angle de champ, l'erreur moyenne de la coordonée "y" étant toujours plus grande que celle de la coordonnée "x". Les résultats optenus ne permettent toutefois pas de dériver un modèle mathématique plausible.

Le type de chambre aérophotogrammétrique qui offre le plus de précision ne peut pas être nettement défini. Le test prouve certes que les chambres super-grand-angulaires procurent une erreur moyenne minimale sur la position des points (x,y,z), vu que leur précision altimétrique est indiscutablement la meilleure et que leur position planimétrique est très légèrement inférieure à celle des autres types de chambres. Les chambres super-grand-angulaires ont cependant une qualité d'image qui se situe audessous de celle des autres chambres et leurs performances s'avèrent défavorables pour les travaux d'aérotriangulation. Pour disposer d'une exactitude planimétrique optimale, on pourrait accorder la préférence aux chambres normal-angulaires, car leur erreur de coordonnées moyenne en "x" et "y" a la plus faible composante irrégulière. La composante systématique de cette erreur compromet toutefois la supériorité des chambres normal-angulaires. Dans cet ordre d'idée, l'élimination des erreurs systématiques prend une importance particulière.

Resumen

A base de vuelos con cuatro câmaras diferentes sobre un campo de pruebas y la orientación analitica subsiguiente de modelos sueltos se examina empîricamente la influencia del ângulo de abertura de la câmara fotogramétrica en la exactitud altiplanimétrica de restituciones fotogramétricas. Los resultados se represen-tan en función del ângulo de la imagen, se discuten y se comparan con resultados ya existentes. Además, se examina un modelo matemática del Dr. MEIER que indica la exactitud como función del ângulo de la imagen. Finalmente, de los errores medios altimplanimétricos se calcula la componente sistemática y se considera la componente irregular para las consideraciones siguientes.

En principio se demuestra que, como era de esperar, el error medio altimétrico disminuye continuamente a medida que aumenta el ângulo de la imagen, mientras que la exactitud planimêtrica es prâcticamente independiento de dicho ângulo. El error medio es prâcticamente independiento de dicho ângulo. El error medio de la coordinada "y" es siempre mayor que el de la coordenada x. Sin embargo, no es posible deducîr de los resultados un modelo matemático plausible.

Tampoco se puede contestar claramente la pregunta por el tipo de câmara aérea más exacto. Aunque la prueba muestra que la câmara supergranangular proporciona el menor error medio de punto (x,y,z), ya que es claramente la más exacta en altura y sólo algo menos exacta en la planimetrîa que los otros tipos de câmara, da una calidad algo peor de la imagen y resultados menos satisfactorios en la triangulación aérea. Para las exigencias más elevadas en cuanto a la exactitud planimétrica podrîa darse preferencia a la câmara de ângulo normal, ya que ella presenta la menor componente irregular del error medio de coordenadas x e "y". Sin embargo, teniendo en cuenta la componente sistemática, la câmara de ângulo normal ya nos es superior. Por este motivo, la eliminación de los errores sistemáticos es de especial importancia a este respecto.