

On Trajectory Determination for Photogrammetry and Remote Sensing: Sensors, Models and Exploitation

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ABSTRACT

This paper attempts to describe the general state of the art of trajectory determination for airborne and terrestrial photogrammetry and remote sensing for mapping. We discuss the need, applications, technology, models, estimation methods and exploitation of trajectories. The paper also summarizes the research done in the past, at the former Institute of Geomatics and currently in GeoNumerics.

1. INTRODUCTION: TRAJECTORY DETERMINATION

Trajectory determination has been with us for many more years than we use to believe. In photogrammetry and remote sensing (PARS), we often associate trajectory determination to INS/GNSS integration for sensor orientation and therefore overlook that traditional sensor orientation is nothing else than trajectory determination at low frequency at unequally spaced time instants (INS stands for Inertial Navigation System, and GNSS for Global Navigation Satellite System). However, it suffices to recall the origins of bundle adjustment and of its application to satellite orbit determination with ballistic cameras (Schmid, H. H. (1974)), to realize how close orientation and trajectory determination are.

This said, with the exception of frame cameras – yes, we know, they dominate the market, – the rest of imaging instruments in our field, have been designed with the assumption that a sufficiently precise and accurate trajectory – at least of the time-Position-Attitude type – is available. In particular, this is the case of the instruments for mobile mapping – terrestrial and aerial – like the laser scanners, the radars and the array cameras. The acquisition frequency of some of the latter is so high that the trajectory “consumer” expects that for any given time t , orientation elements $o(t)$ be extracted of the trajectory τ that fulfil its – of the sensor – geometric quality requirements. (Today, for our mapping imaging instruments, the interval of acquisition frequencies is, in terms of orders of magnitude, $]10^6, 0[$ Hz.)

We review here some concepts. Roughly speaking, a trajectory is the path described by a point that moves in some given space. Trajectory determination is the estimation of a trajectory's path from measurements and a priori knowledge, e.g. physical laws and mathematical models. Mathematically speaking, given a navigation space, we can consider deterministic and stochastic trajectories, being the latter, the ones of interest. Thus, let $[t_0, t_e] \subset \mathbb{R}$, and let $n \in \mathbb{N}$, $n > 0$; we define a n -dimensional stochastic trajectory τ as a n -dimensional stochastic process on $[t_0, t_e]$. We will refer to $\tau(t)$ as the state vector of τ for any $t \in [t_0, t_e]$. We will assume that our trajectories are continuous; i.e., that for any $t \in [t_0, t_e]$ there is $\tau(t)$, a random variable or, that if τ is a discrete stochastic process, there is an associated interpolation method to estimate $\tau(t)$. In the latter case of discrete processes, the known state vectors $\tau(t_0), \dots, \tau(t_e)$ are called the trajectory anchor points. When it makes sense, we will also refer to the frequency of the series t_0, \dots, t_e as the trajectory frequency.

For the purpose of serving the current $]10^5, 0[$ Hz imaging instruments band, trajectory determination systems are designed in a way that the primary motion-sensing instrument is an inertial measurement

unit (IMU) and the secondary one is a GNSS receiver. Depending on the application and the context, other auxiliary instruments – a.k.a. aiding ones – are included in the design like distance measurement instruments (DMI) – the *odometers* – or highly stable atomic oscillators – the chip-scale clocks. We will refer to these motion-sensing instrument configurations and methods as INS/GNSS-based systems and methods respectively. Thus, while INS/GNSS-based systems take responsibility for the $]10^5, 1[$ Hz imaging frequency band, inverse imaging-based methods, like the bundle adjustment, can absorb the remaining low frequency band with image sequences, like *video* sequences, that can deliver trajectories in the $]10^2, 0[$ Hz order of magnitude trajectory frequency band. More precisely, inverse imaging-based trajectory determination refers to trajectory determination dominated by the use of measurements derived from the PARS payload instruments.

One advantage of inverse imaging-based methods is that the trajectory anchor points coincide with the expected trajectory queries $\tau(t)$ and frequency is not an issue in this case. (Contrary to what most of us tend to think, the required trajectory frequency mainly depends on sensor carrier vehicle dynamics rather than on sensor features, simply because we want to recover a continuous trajectory from interpolation.)

Having now introduced the concepts of INS/GNSS-based and inverse imaging-based methods, we can say that in some scenarios the use of either one of the methods is clear (they “specialize”), in some other scenarios is unclear (they “compete”) and in other scenarios, the combination of the two methods is unavoidable (they “cooperate”). Note that the scenario is defined not only by the instrument configuration and the environment, but also by the trajectory exploitation requirements, since demanding requirements may result in the need of making all sensors and methods cooperate. Last, we note, that in the past times of monosensor geodata acquisition, the determination of the carrier trajectory had to be precise because all what we needed was a consistent set of orientation elements that would generate accurate geometric information on ground. Today, a trajectory has to be precise and accurate, because multisensor acquisition systems cannot tolerate consistent trajectories for single sensors that inconsistent across multisensor systems. In the future, depending on the application – like real-time navigation or geoinformation for critical tasks – a trajectory will have to be precise, accurate and extremely reliable.

The paper is organized as follows: section 2 briefly summarizes the currently used measurement techniques for trajectory determination in RPAS; section 3 discusses motion sensing and timing technology; section 4 summarizes the modelling and estimation aspects and section 5 presents some examples of non-standard trajectory exploitation techniques.

2. ON NECESSARY COOPERATION FOR SELF-CALIBRATION

As we already pointed out, since the late eighties most new mapping paradigms have been built upon sensors and acquisition modes that require INS/GNSS-based systems like the array cameras (1D geometry) even if they are of the multi-array type, the Interferometric Synthetic Aperture Radar (InSAR), the laser scanners (0D geometry) and the terrestrial mobile laser scanner and camera systems. In some of these cases, even though INS/GNSS-based methods may be sufficient for trajectory determination and orientation transfer to the PARS payload – direct sensor orientation – the instabilities and errors of the PARS instruments may ask for self-calibration techniques which, in turn, will ask for the combination of INS/GNSS – and inverse imaging-based methods – integrated sensor orientation. Table 1 provides a representative, not necessarily comprehensive, summary of some current common and experimental sensor configurations for trajectory determination and sensor self-calibration.

	INS	GNSS	DMI	AC	tie points	tie lines	tie planes	GCPs
airborne InSAR	x	x		+				+
airborne laser	x	x			x	x	x	x
airborne array cameras	x	x			x	x		x
airborne frame cameras	x	x			x	+		x
airborne frame cameras (UAS)		+			x			x
terrestrial mobile mapping	x	x	x	+	x	+	x	x

AC: atomic clock. x: usually included. +: starts being used or considered.

Table 1: Type of technology or measurements involved in trajectory determination and sensor self-calibration for RPAS.

3. PROGRESS IN MOTION SENSING AND TIMING

3.1. GNSS Infrastructure

Since the practical advent of GPS (1995, Full Operational Capability) and GLONASS (1995 constellation completion), two major developments have taken place in satellite-based navigation: the availability of complementary, material and immaterial, augmentation infrastructures – like the International GNSS Service (IGS) or the satellite based augmentation systems like WAAS (USA) and EGNOS (EU) – and the birth of Galileo and BeiDou.

Thus, for instance, today, with the mentioned infrastructure, with dual-frequency geodetic-grade receivers and calibrated antennas, it is possible to estimate GPS-based trajectories at the cm level precision and dm level accuracy when using the Precise Point Positioning (PPP) method – an Earth global, non-differential approach – introduced in the seminal article (Zumberge, J. F., et al. (1997). Today, triple-frequency PPP investigations are already under way (Laurichesse, D. and Langley, R. (2015)).

The IGS is providing GPS satellite orbits with 2.5 cm errors (error vector norm, RMS) and clocks with 75 ps accuracy and 20 ps precision – i.e., 75×10^{-12} s and 20×10^{-12} s or 2,25 cm and 0,6 cm respectively. Additionally, final ionospheric TEC (Total Electron Content) grids with an accuracy of 2 to 8 TECU (TEC Units) are also provided at temporal sampling intervals of 2 h and spatial sampling pixels of 5 deg (longitude) \times 2.5 deg (latitude) (Dow, J. M. et al. (2009) and IGS web page (IGS (2015)).

By 2020, the number of navigation, positioning and timing satellites will be doubled thanks to the contribution of the European (EU) and Chinese, Galileo and BeiDou systems respectively. Galileo has been designed to be compatible and interoperable with GPS. Already now, many geodetic-grade receivers and antennas are prepared to receive signals in the L1, L2, L2C, L5 GPS; E1, E5a, E5b, E5 AltBOC Galileo, L1, L2, L3 GLONASS and B1, B2 and B3 BeiDou frequency bands. This makes up to receivers with more than 800 tracking channels. Typically, a receiver board – a.k.a. an OEM receiver – weighs about 80 g and has a surface of some 9×9 cm². These physical characteristics together with a moderate power consumption (8 W) make top performance GNSS receivers amenable for virtually any PARS payload carrier including unmanned aircraft.

The IGS is already preparing itself for the coming 4-constellation scenario and has set-up the Multi-GNSS Experiment (MGEX) to track, collate and analyse all available GNSS signals (Montenbruck, O. et al. (2014)).

The large number of satellites – more than 100 – and measurements will certainly improve the precision, accuracy and reliability of tPV determination. However, it is not only that we will have just more satellites but also that some of the signals and modulations are of an unprecedented ranging precision and anti-multipath properties. This is the case of the new ranging codes of the GPS L5 signal ($\sigma_p \approx 4\text{-}6$ cm) and, particularly, of the Galileo E5 AltBOC ($\sigma_p \approx 2$ cm) which, in open sky conditions (Colomina, I. (2012)), allows for accurate navigation and positioning ($\sigma_H \approx 0.07$ m and $\sigma_V \approx 0.19$ m). The high precision of the Galileo E5 AltBOC pseudoranges makes them ideal for the single-frequency GNSS ionospheric delay estimation (Schüler, T., Abel-Oladipo, O. (2012) and Schüler, T. (2012)), an already existing concept (Xia, R. (1992)) negatively affected by the high noise of the GPS L1 pseudoranges. Single-frequency ionospheric delay estimation has a positive impact in the cost of GNSS receivers and on the performance of dual- or multi-frequency professional receivers.

The E5 AltBOC signal exhibits the smallest multipath error ever seen in satellite navigation signals (0.3 m in the worst case).

If we consider the aforementioned scenario, it is not unrealistic to foresee a sub-cm-level and cm-level, horizontally and vertically respectively, accuracy GNSS-based trajectory determination at a global scale without the direct use of local augmentation infrastructure.

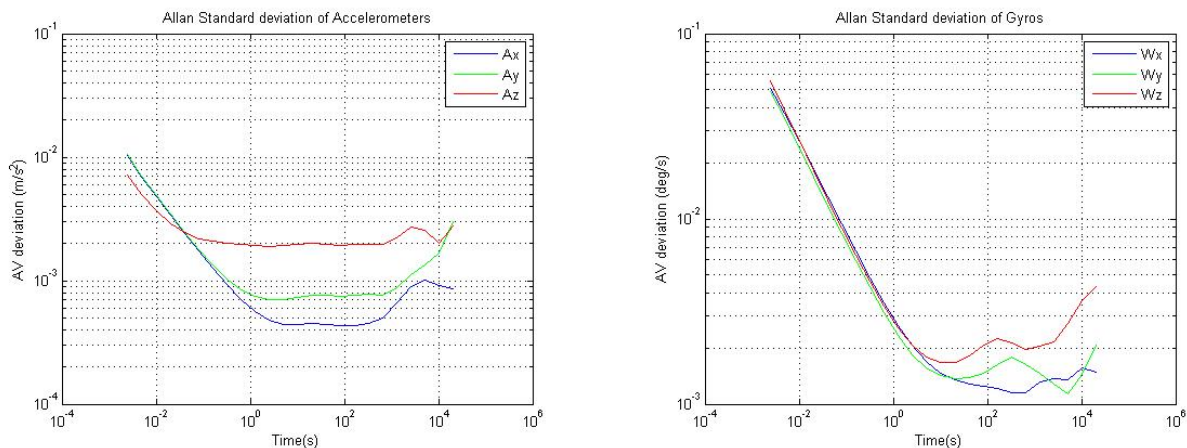


Figure 1: Allan Deviation for the Seiko's Epson M -G350-PD11 (accelerometers and angular rate sensors).

3.2. Inertial sensing and navigation

Inertial sensing refers to the measurement of physical signals produced by accelerating and/or rotating bodies with respect to an inertial frame of reference. The two most popular types of inertial sensors are the linear accelerometers and the angular rate sensors. For traditional PARS, the latter are usually of the Fibre Optic (FOG) type; i.e., FOGs with more than 1 km coil length have practically replaced the Ring Laser Gyros (RLG) and FOGs with a few hundred meters coil are commonplace. Another available family of angular rate sensors up to now confined to space applications is the Hemispherical Resonator Gyros (HRG) which thanks to new design and manufacturing concepts starts to challenge the FOG family (Rosellini, L. and Caron, J. M. (2008)) and (Jeanroy, A., et al. (2014)). Micro Electro-Mechanical System (MEMS) technology for linear accelerometers is frequently used together with the FOGs and the HRGs.

Since inertial sensors are related to navigation and orientation parameters through stochastic differential equations, in the [numerical] integration solution process, the measurement errors result in positional, velocity and attitude cumulative errors – the drifts – that, to a large extent, can be corrected with other measurements, most times GPS carrier phases and pseudoranges or GPS-derived positions. Today, inertial sensors are the primary measurement instruments of many kinematic surveying systems: they provide the high frequency observation stream for the computation of the tPVA (time-Position-Velocity-Attitude) trajectory solution whereas the other [secondary] instruments provide lower frequency observation streams to mitigate the inertial sensor errors.

One of the beauties and advantages of inertial technology for trajectory determination is its broad range of quality and associated costs. An inertial measurement unit for PARS can cost between 150 k€ and 5 €. Correspondingly, its applications can range from accurate direct sensor orientation to a simple GNSS-based trajectory interpolator – higher frequency – and extender – velocity and attitude.

The quality of an IMU is primarily a function of its accelerometers and its angular rate sensors. However, in practice, the finest results are not necessarily obtained by the numerical integration of high-quality inertial measurements but from the balanced integration of an IMU and a GNSS receiver measurements, the so-called INS/GNSS integrated systems. If an IMU is designed for INS/GNSS integration then, once an angular rate technology has been selected – like RLG, FOG or HRG – the performance of the INS/GNSS system is finally tuned by the quality of the linear accelerometers to the point that for a fixed, given high-quality angular rate sensor, the IMU export licenses – related to defence and security issues – depend on the linear accelerometer quality.

Today's IMUs for PARS are dominated by the following configurations: (FOG, MEMS) and (MEMS, MEMS) where the former stands for an IMU with FOG angular rate sensors and MEMS linear accelerometers, and the latter for MEMS angular rate sensors and MEMS accelerometers. Probably, the most representative IMU of the (FOG, MEMS) family is Northrop Grumman's LN-200 a tactical-grade IMU – following the traditional IMU classification grades: navigation, tactical and low-cost. The (MEMS, MEMS) family is well represented by the Seiko's Epson M-G350-PD11 – a 2 k€, 7 g, $24 \times 24 \times 10 \text{ mm}^3$ piece of hardware – that exhibits a remarkable run-to-run stability and low noise behaviour (see figure 1). It is remarkable, that some of the noise figures of the Epson are similar to those of the even lower cost IMU, the InvenSense MPU 6500 – a 5 €, negligible weight, $2.9 \times 2.9 \times 0.9 \text{ mm}^3$ tiny instrument– for video games and other mass-market applications. Table 2 summarizes the features of these IMUs. We note that the run-to-run bias, once it is estimated, it stays as constant as the in-run stability figure dictates. On the other side, once the run-to-run bias is captured, the performance of the Epson and Invensense IMUs are comparable.

	angular rate sensors			linear accelerometers		
	noise	run-to-run stability	scale factor error	noise	run-to-run stability	scale factor error
	deg/h/sqrt(Hz)	deg/h	ppm	$\mu\text{G}/\text{sqrt}(\text{Hz})$	μG	ppm
Norhrop-Grumann LN-200e	3	0.5	100	35	300	300
Epson M-G350-PD11	12	1800		100	800	
InvenSense MPU 6500	30	18000		100	6000	

Table 2: Noise, repeatability and scale factor accuracy for tactical, high-end MEMS and consumer MEMS IMUs.

In recent times, however, important as the parameters in table 2 may be, the quality of IMUs is rigorously and commonly described with its Allan Variance (AV), a function of time, or more to the point of averaging times, that provides the root mean square (RMS) random-drift errors. With the exception of run-to-run variations, the AV is a sound way to characterize IMUs. The AV is a time-domain – as opposed to frequency-domain – analysis technique originally developed to understand the stability of precision oscillators. Because of the somewhat similarity of oscillator and inertial sensor errors, the AV has been massively adopted for the analysis of inertial sensors' quality. As indicated, figure 1 presents a graphical representation of the AV for the Epson M-G350-PD11 and table 2 provides comparative figures among the LN-200, the Epson and the InvenSense. Also, as indicated, considering the price difference, the specifications (table 2) are remarkably similar.

With the aforementioned recent developments in sensing miniaturization and performance, the old classification navigation-, tactical-, automotive/industrial- and low-cost/consumer-grade does not make much sense as most IMUs above could be called “tactical.” Moreover, in addition to the AV, since the position and velocity (tPV) is dominated by GNSS, the IMU performance in terms of navigation can be described in simpler ways. This is an open question with one of the possible answers being the heading precision/accuracy and drift after coarse self-alignment at a predefined latitude for some predefined period of time.

Last, we envision a future strategy of combining multi-constellation multiple-frequency geodetic-grade GNSS receivers with high-, medium- and low-grade tactical IMUs. As of today, performances for (FOG, MEMS) and (MEMS, MEMS) IMU configurations are summarized in table 3.

	mode	P (m)	V (m/s)	A(r,p) (deg)	A(h) (deg)
(FOG, MEMS)	RT	0.04 – 0.08	0.008	0.004 – 0.010	0.012 – 0.030
(FOG, MEMS)	PP	0.02 – 0.05	0.005	0.003 – 0.008	0.008 – 0.015
(MEMS, MEMS)	RT	0.04 – 0.08	0.03 – 0.04	0.04	0.20
(MEMS, MEMS)	PP	0.02 – 0.05	0.01 – 0.02	0.03	0.10

RT: real-time or forward processing. PP: post-processing or forward/backward processing. r, p, h: roll, pitch, heading.

Table 3: Performances of (FOG, MEMS) and (MEMS, MEMS) units for INS/GNSS integration.

3.3. Timing

Timing is an important, sometimes hidden and/or forgotten aspect, of trajectory determination. An accurate trajectory whose time reference frame is different from the one used by a sensor results in a synchronization error and, subsequently in a spatial error (Blázquez, M., Colomina, I. (2012b)). However, beyond this obvious issue of consistent time reference frames or correct “time-tagging,” there is a well-known interdependency of positioning and timing in GNSS-based trajectory determination, where the four fundamental unknowns in the GNSS range equations are three spatial coordinates and one temporal one. Any instrument that can generate time measurements or equivalent data, like a high quality stable frequency oscillator, makes trajectory determination more robust against GNSS signal obstruction, multipath or even jamming/spoofing. More specifically GNSS receiver clock errors are highly correlated to the height and height-related errors. In this respect, the recent introduction by Symmetricon, in 2011, of the first chip-scale atomic clock (CSAC) with a weight of 35 g and a moderate cost (about 2 k€) has been a breakthrough, still not sufficiently acknowledged and leveraged. On the accuracy side (Krawinkel, T., Schön, S. (2014)) report improvements in the height position and velocity of about 60%. On the robustness side, the use of a

CSAC is of clear interest in terrestrial mobile mapping. In the meantime, since 2011, other vendors sell variants of the CSAC technology.

4. TRAJECTORY ERROR MODELING AND ESTIMATION

4.1. Modelling

Trajectory error modelling is the key to quality trajectory determination. As in any other estimation problem, the “better” the model is, the more significant the improvement will be and the lesser the required additional measurements. Trajectory error modelling is approached in two ways: (1) modelling of the errors at the trajectory level and (2) modelling of the errors at the sensor level. Generally speaking modelling of sensor errors and particularly of MEMS sensors is an open and challenging question (Waegli, A. et al. (2010)).

Modelling of the errors at the trajectory level is an option when there is no access to raw inertial and satellite range measurements or when there is no knowledge about the corresponding mathematical models. In airborne PARS, the simplest and most popular trajectory error model is the shift correction, per strip or per block, in integrated sensor orientation. This model has now been in use for 25 years and continues to work well at least for non multisensor systems. Reason for the success of the shift-correction model – a 0-degree polynomial – is that for short to moderate time periods the change in GNSS trajectory induced errors is small. However, in less friendly GNSS environments, like those of terrestrial mobile mapping, the pattern of GNSS errors due to obstructions, signal attenuation and multipath is far more complex and accuracy/consistency errors of 0.5 m and larger are common. Therefore, ground control points (GCPs) are regularly used in terrestrial mobile surveys. In view of this complexity and of the availability of GCPs, higher degree polynomial error models with continuity constraints or splines have been introduced recently. First results with these models indicate that they can mitigate part of the problem but not solve it in the sense of meeting mapping specifications. Other attempts to model trajectory errors have been made with stochastic error processes – like random walks or 1st order Gauss-Markov processes with short correlation times – but results are not yet available.

Current GPS is definitely not sufficient to guarantee correct trajectories in urban and semi-urban environments. Since the use of GCPs is an expensive and time consuming alternative, the integrated sensor orientation method including inverse imaging-based models with laser scanner and photographic cameras seem to be the solution (Angelats et al. (2014)). Note, that GCPs and measurement of tie-features like points, segments or planar surfaces (Angelats et al. (2012)) are measurements and not models and that they can be used at the trajectory level or at the sensor level modelling like in (Soloviev, A. (2012)).

As for the error modelling at the sensor level, for IMUs, odometer and other motion sensors there is no difference with respect to the sensor calibration models used in INS/GNSS the only difference being that GNSS measurements are replaced by other ones.

4.2. Estimation

Trajectory estimation is an active field whose more important dimensions are, to our best knowledge, the following: (1) context aware vs. context unaware estimation, (2) integration levels (INS/GNSS loose, close, tight and deep coupling), (3) standard prediction-filtering-smoothing techniques vs.

other stochastic filtering techniques and (4) navigation approach vs. geodetic approach and state-space approach vs. dynamic-network approach.

Being aware of the navigation context is, many times, more important than the mathematical models and estimation approach (Groves et al. (2014a)) and (Groves et al. (2014b)) as, from context, we can select the appropriate models and estimation method thus avoiding the main risk of the Kalman filtering method, filter divergence or convergence to local minima. Since this is almost an artificial intelligence problem, intermediate techniques, based on a priori information, (Parés, M. E. and Colomina, I. (2015)) have been proposed for automated estimation strategies.

Integration levels (Silva et al. (2007)) at the close level (using the raw GNSS ranges in the Kalman filter update step) and tight level (using the predicted Doppler shift as feedback from the Kalman filter to the GNSS receiver tracking loops) are nowadays preferred over the simple loose level (using GNSS point coordinates as update measurements) and over the more complex deep coupling as it requires the processing, in the navigation filter, of the early, prompt and late (E, P, L) of the inphase (I) and quadrature (Q) signals. Taking into account the code ranging and anti-multipath features of the next generation GNSS signals, PARS applications are not asking for this level of complexity in INS/GNSS data processing.

In our field, the state of the art in INS/GNSS and, in general, multisensor-based trajectory determination is the Kalman filter (KF) – formulated 55 years ago (Kalman, R. E. 1960) – and, more specifically, the Iterated Extended Kalman Filter (IEKF) with its many variants. It is an extended misconception that one of the KF hypotheses is that the measurements and unknown parameter errors are Gaussian distributed. It is also a misconception that the KF approach is restricted to linear models as non-linear models can be processed with the IEKF extended method. However, strongly non-linear models and strong departures from Gaussian distributions call for other type of methods. One of the successful alternatives is sequential Monte Carlo estimation (Ristic, B. et al. (2004)) known as particle filters. An interesting example to navigation and positioning in the context of multipath environments is provided in (Closas, P. et al. (2010)). For the time being, though, the IEKF has proven to be sufficient for most trajectory determination problems in PARS.

Geodetic approaches to trajectory determination, with emphasis on post-mission processing, have been discussed in (Sansò, F. (2006)) and (Colomina, I., and Blázquez, M. (2004)). In the latter reference, multisensor trajectory determination is modelled with stochastic equations, $g(l + v, x) = 0$ (a.k.a. observation equations in geodesy) and with stochastic differential equations, $g(l + v, x, \dot{x}) = 0$, that can be regarded as a generalization of the classical observation equations to differential relationships. With this approach, the traditional Kalman forward, backward and smoothing method can be replaced by a geodetic network adjustment once the x parameter derivative, \dot{x} , is replaced by a numerical approximation. This technique has already been used in INS/GNSS strapdown airborne gravimetry in research projects. It has the advantage that, as opposed to Kalman-based sequential estimation, trajectory cross-overs can be dealt with as if they were network “tie points” to improve the geometry of the problem. We also note, that with this technique, outlier detection and removal can be performed as usual with geodetic regression diagnostic (data snooping) techniques.

5. PROGRESS IN THE EXPLOITATION OF TRAJECTORIES

The potential if INS/GNSS-based trajectories is many times underused; either because not all trajectory information is used or because the trajectory error properties are not fully understood. In the next sections, we illustrate this with results of our own research.

5.1. Exploitation of the error properties

Relative aerial control. In (Blázquez, M., Colomina, I. (2012a)) we demonstrated the advantages of decorrelating INS/GPS-based aerial control by using relative position-attitude observations. Based on the knowledge that INS/GPS-derived trajectories are strongly auto-correlated – in the sense of a stochastic process, – relative measurements between consecutive images provide a data set of highly stochastic independent observations. Apart from the advantages of a higher fidelity stochastic model and moderate improvements in the results, the hypothesis of outlier detection with regression diagnostic methods are fulfilled and confirmed in practice.

Simplification/elimination of the tie point search and measurement. In (Blázquez, M., Colomina, I. (2012c)) and (Blázquez, M., Colomina, I. (2012d)) we demonstrated the high level of precision and short-term accuracy of INS/GPS-based trajectories by removing all tie points image measurements from an integrated sensor orientation (ISO) adjustment except those being a Ground Control Point (GCP). The combination of this particular measurement configuration with the ISO method was called FAST AT.

FAST AT delivered results much closer to standard ISO than to direct sensor orientation (DiSO) – a.k.a. direct georeferencing – thus indicating that image processing for tie point generation is an option.

Kinematic ground-control points. In (Molina, P. et al. (2015)) we introduced the concept of the Kinematic Ground Control Point (KGCP); i.e., a GCP materialized with a visible target on a vehicle. We also introduced the pointing-and-scale photogrammetric measurements of the KGCP by taking advantage of the target size. In this way, in a tandem terrestrial and aerial mapping mission, two trajectory information sets from the aerial and from the ground vehicle, can be combined in an unprecedented geometrically strong and consistent relative manner allowing for the self-calibration of a camera interior orientation elements, radial and tangential distortions.

(The concept has been applied to combined terrestrial mobile mapping and unmanned aircraft mapping and has been awarded the EU H2020 mapKITE, ref. [641518](#), research and development project.)

In the KGCPs we are exploiting the high frequency contents of INS/GNSS-based trajectories and their local consistency –i.e., correlation– due to the eventual common GNSS remaining errors.

5.2. Exploitation of the full content

Time-space calibration. In (Blázquez, M., Colomina, I. (2012b)) we demonstrated 4-dimensional simultaneous time-space calibration down to the 0.1 ms precision level, by using INS/GPS-based full aerial control (time-position-velocity-attitude) and extended aerial control (time-position-velocity-attitude-angular velocity). As opposed to the common practice, we introduced the use of linear and angular velocities of the camera, as obtained and transferred from the tPVA solution of an integrated INS/GPS system. By using the available linear and deducible angular velocities, we were able to

transform time errors into spatial ones and, thus, use the spatial redundancy of a bundle adjustment for the calibration of synchronization errors between the navigation and imaging components of multisensor system.

Exploitation of the linear velocities and of the calibrated angular rates. In (Colomina, I., Blázquez, M. (2014)) we proposed to extend the concept of image exterior orientation (tPA) to the image state, time-Position-Velocity-Attitude-Angular rates (tPVAW) with applications to image deblurring, time-space calibration and the accurate orientation of low-cost cameras via the modelling of the focal-plane shutter behaviour.

6. CONCLUSIONS

Trajectory determination is one of the fundamental techniques of photogrammetry and remote sensing (PARS). The techniques and methods of trajectory determination origin from different disciplines: PARS itself, image processing and computer vision, geodesy, inertial navigation, satellite navigation, timing and mathematical optimal estimation. We are witnessing a period of formidable development in practically all the just mentioned topics. It is only up to our PARS community to know and optimally combine these tools for better serving our economy and society.

7. ACKNOWLEDGEMENTS

The author gratefully acknowledges the discussions and data provided by M. Eulàlia Parés (Division of Geomatics, CTTC).

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