From Bohnenberger's Machine to Integrated Navigation Systems, 200 Years of Inertial Navigation

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ABSTRACT

In 1817, F. Bohnenberger presented an apparatus consisting of a fast spinning rotor with cardanic suspension. His invention formed the basis for L. Foucault's epochal studies on gyroscopic sensors in the middle of the 19th century. The development of instruments like the artificial horizon and the directional gyro is a direct result of this work. Later on, gyro technology led also to stabilized platforms forming the early variants of inertial navigation systems.

Foucault's initial intention of using Bohnenberger's invention as a sensor was the provision of a device offering an inertial direction reference for surveying and navigation. Due to technical limitations, a gyro is, however, suitable for such a task during only a limited period of time. To enable a long-term usage, additional, complementary measurements are required. An artificial horizon therefore includes levelling sensors, and a directional gyro is typically coupled with a magnetic compass. On the other hand, the main feature of integrated navigation systems is the fusion of data from dissimilar signal sources. Thus, artificial horizons and directional gyros represent elementary forms of such systems.

Modern integrated navigation systems of high performance consist of the combination of inertial strapdown systems (being the successor of stabilized platforms) and GPS receivers. In photogrammetry, they have become an essential tool for direct georeferencing. Nevertheless, their system structure is still traceable to the early gyro instruments mentioned. This classical system design being outlined in the following opens perspectives for new developments.

1. HISTORICAL DEVELOPMENT OF GYROSCOPIC INSTRUMENTS

Working on a simple experimental proof for the rotational motion of the earth, the French physicist Léon Foucault introduced in 1852 the term *Gyroscope* for an instrument being able to observe such movements (Foucault, 1852). Besides his well-known pendulum, these studies concentrated on gyros with cardanic suspension (Figure 1). Foucault recognized especially that a well-directed restraint of the motion of the gimbals (like blocking one degree of freedom of the suspension) leads to specific indicators or sensors detecting different rotation components (Broelmann, 2002). With that, he paved the way for such important gyro instruments like the artificial horizon, the directional gyro, and the gyrocompass (Sorg, 1976). Moreover, this was also the basis for the development of stabilized platforms (Wrigley, 1977) leading into modern inertial navigation systems.

L. Foucault is, however, not the originator of gyros with cardanic suspension. He was familiar with this mechanical principle because such instruments were already employed in many French schools to explain the precession of the earth rotation axis. This matter was born of the initiative of the French mathematician Pierre-Simon Laplace, who referred likely to an initial specimen of the École Polytechnique in Paris (Poisson, 1813). The inventor of this device was J.G. Friedrich Bohnenberger (1765 – 1831), a former Professor for mathematics, astronomy, and physics at the University of Tübingen, Germany. Using the drawing shown in Figure 1, he explained retroactively but for the first time the design and the use of gyros with cardanic suspension (Bohnenberger, 1817). As F. Bohnenberger could not yet know the term gyroscope, he called the device simply Machine. Therefore, the *Machine of Bohnenberger* was the basis for Foucault's epochal work on gyros.[1]

The initial intention of using a gyro as a sensor was the provision of an instrument offering a self-contained direction reference for surveying and navigation. This idea was born even before Boh-

[1] Following today's terminology, F. Bohnenberger was geodesist. His scientific work was dedicated mainly to surveying and cartography, and he is regarded as the founder of the Surveying Authority of Württemberg in the southwest of Germany, too. This fact illustrates how closely geodesy and gyro technology are traditionally connected.

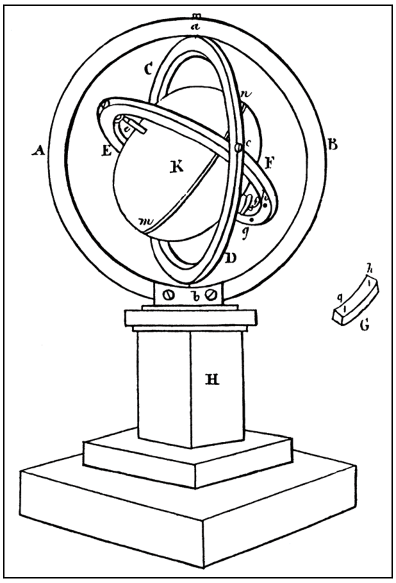

Figure 1: Bohnenberger's original drawing of a gyro with cardanic suspension (1817).

nenberger's invention: In 1742 or 1743, the Englishman John Serson presented a fast spinning top, whose upper surface perpendicular to the axis of its rotation was a circular plate of polished metal. When the top was set in motion, the plain part of its surface became horizontal while maintaining this behaviour even in a disturbing environment like on swaying ships. With this, J. Serson proposed a solution of the problem of finding a satisfactory horizon for use in sextant observations at sea when there was fog around the sea horizon. Herefrom the artificial horizon originates.

Unfortunately, J. Serson lost his life in 1744 during a test campaign at sea. It took about 140 years before his invention was revived in France by Admiral G. Fleuriais. In the meantime, not only L. Foucault made important contributions to gyro technology. The problem of keeping the rotor at an approximately constant speed became better controllable by using electrically or pneumatically driven motors. When people began furthermore to construct and to fly airplanes during the following decades, it became necessary also for the pilots to have an artificial horizon in clouds and during the night. The First World War stimulated particularly the technical progress in this area, and in 1930 the American Elmer Sperry succeeded in designing an especially reliable instrument. It was an air-driven artificial horizon based on a gyro with cardanic suspension and with controlling the vertical position of the gyro axis by pendulums and air jets (Sorg, 1976).

Besides the artificial horizon, the next important basic type of gyroscopic instruments is the directional gyro. Referring to Figure 1, F. Bohnenberger (1817) described already its main property (translated analogously): "While the rotor is spinning around its axis, this axis will maintain permanently that direction which was given to it at the beginning. This will happen well then if one takes hold of the pedestal of the apparatus and starts moving it. While carrying around the Machine, one can move in arbitrary directions and with arbitrary velocities, and the axis of the sphere will permanently remain parallel to itself and will permanently stay aligned to north like a magnetic needle if one has, for example, orientated it at the beginning to north."

Due to technical limitations like friction in the gimbal bearings, the preservation of the rotor axis alignment is possible only for a limited period of time. To enable a long-term usage, the orientation of the gyro has to be aided suitably. Besides the use of a magnetic compass as described below, an unbalanced inner gimbal of the cardanic suspension proved to be helpful in this connection (Magnus, 1971). The latter case led to the gyrocompass and gyro theodolite. These instruments, called north-seeking gyros, sense the direction of the earth rotation vector and align themselves thereupon to north. Following a first attempt of L. Foucault to build such a device, many scientists participated during the following decades in the development of the gyrocompass. (Hermann Anschütz-

Kaempfe and Max Schuler are known in particular.) An electrically driven rotor, a floated suspension, and the combination of two or three jointly used rotors were important contributions leading to reliably usable models at the beginning of the 20th century.
Reconceived by means of modern control theory, the artificial horizon, the gyrocompass or gyro theodolite, and the directional gyro aided by a magnetic compass represent special realizations of observers. This aspect enables to classify these instruments in a more general way leading directly to inertial navigation systems and their aiding. For this, section 2 illustrates next the observer principle by the example of integrating a directional gyro and a magnetic compass. Section 3 contains subsequently the transition from single gyro instruments to inertial navigation systems (INS), and section 4 addresses the special coupling of these systems with GPS satellite navigation receivers. Motivated by still existing performance limitations of the INS/GPS integration, section 5 outlines further development work for such systems. Section 6 finishes the paper with a general assessment.

2. SOME BASICS ON INTEGRATED NAVIGATION

2.1. Integrating a Directional Gyro and a Magnetic Compass

Figure 2 illustrates the principle of aiding a directional gyro with a magnetic compass: The fast spinning rotor with cardanic suspension shows, in principle, an inertially constant orientation. However, if a torquer appropriately readjusts the outer gimbal of the mechanism, the axis can be coupled to the mean motion of a magnetic needle. With this, the integrated instrument has the long-term accuracy like a magnetic compass but shows the much smoother reading of a gyroscopic indicator.

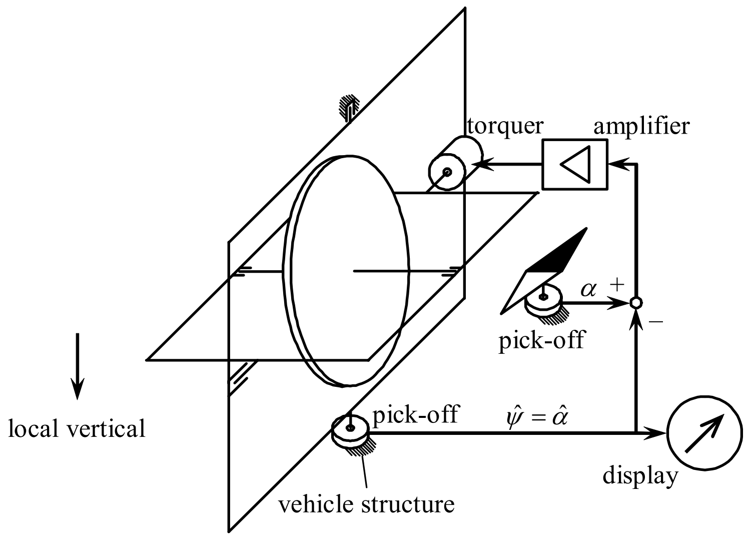

Figure 2: Principle of a directional gyro aided by a magnetic compass.

Figure 3 is a schematic representation of the signal flow of this device (it is very abstract but leads directly to the more general observer principle): A vehicle equipped with such an instrument shall be subject to the vertical angular rate $\dot{\psi}$ leading to a time varying heading ψ. Based on ψ and the swaying of the magnetic compass, the needle pick-off generates the signal α. In parallel to the vehicle, the gyro is also subject to $\dot{\psi}$. It is now possible to deem the rotor and the gimbals as a mechanical computer integrating $\dot{\psi}$ numerically with respect to time. The result of this procedure is the displayed estimate $\hat{\psi}$ of the true heading ψ. It is identical to the estimate $\hat{\alpha}$ generated by the gimbal pick-off. The difference between $\hat{\alpha}$ and the "true" value α forms the input of a series connection of an amplifier and a torquer having the property of a low-pass filter and controlling the alignment of the outer gimbal. With this, the difference between ψ and its estimate remains normally small.

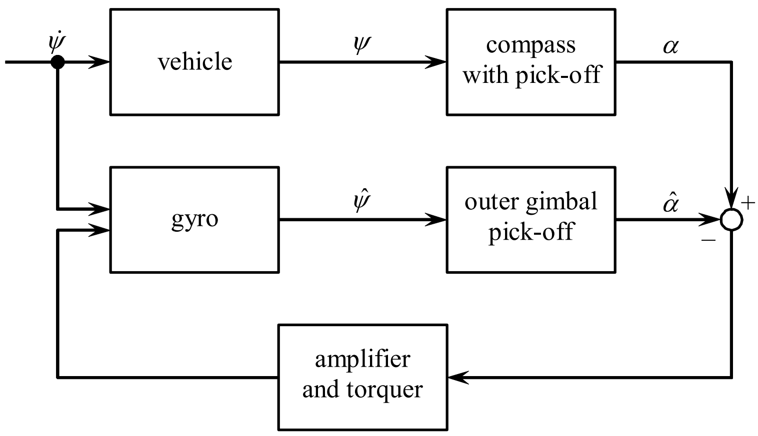

Figure 3: Block diagram for the signal flow of Figure 2.

2.2. The Observer Principle

Despite its simple design, the gyro aided by a magnetic compass is representative for the general properties and system structure used for an extensive class of integrated navigation systems:

- functional combination of dissimilar signal sources with complementary properties (i.e. in particular various measurement principles),
- increased performance compared with the abilities of the components separately,
- observer principle as authoritative integration scheme.

The last of these three points follows from the fact that Figure 3 is simply a special case from Figure 4 representing the general case of a multivariable observer (Levine, 1996): The angular rate $\dot{\psi}$ corresponds to the input $\mathbf{u}$, ψ matches the motion state $\mathbf{x}$ and α the aiding vector $\mathbf{y}$; the compass acts as the aiding sub-system.

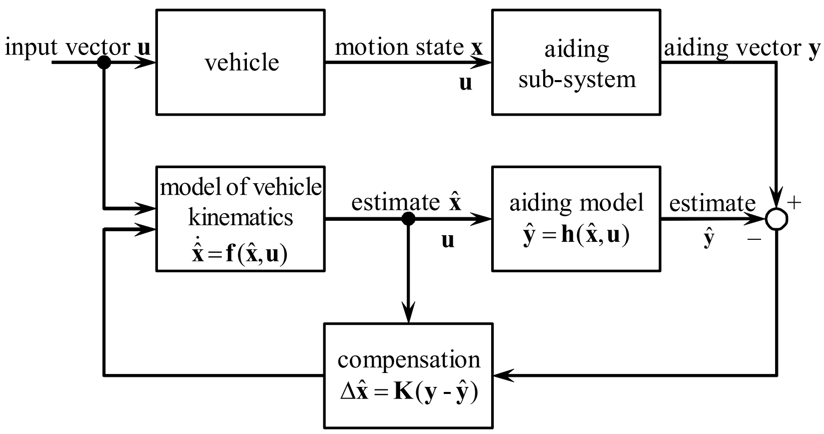

Figure 4: Observer principle employed for integrated navigation systems.

The middle branch in the diagram of Figure 4 is the central point of the observer. It functions as a device reproducing the initially unknown vehicle motion $\mathbf{x}$ (lower left block) and the operation of the aiding sub-system employed (lower right block): A set of ordinary differential equations with vector function $\mathbf{f}$ describes the vehicle kinematics considered by the integrated system, and a set of algebraic equations with vector function $\mathbf{h}$ models the aiding part. To adapt furthermore the estimate $\hat{\mathbf{x}}$ to the real state $\mathbf{x}$, a reasonable design of the compensation matrix $\mathbf{K}$ is essential. Integrated navigation systems employ in particular the Kalman filter theory (Gelb, 1989) for this purpose.

In principle, it is possible in Figure 2 to replace the gyro mounted on gimbals by a vertically aligned rate gyro together with a numerical integrator calculating digitally ψ from the measured rate $\dot{\psi}$.

This simple example shows that the realisation of an observer is not bound to a certain technology. Generally, neither the sensor principles employed nor special mechanical, electrical or software elements realizing the system of Figure 4 make up the character of an integrated navigation system. The main distinguishing feature consists in the vehicle and aiding model used (i.e. **x**, **f** and **h**).

2.3. Motional Degrees of Freedom

To illustrate the last statement, Table 1 contains several traditional variants of integrated navigation systems. They differ primarily in the degrees of freedom considered from the vehicle motion. With this, it is then possible to regard the various sensor types as being simply a result of this classification. ("Which sensor principles are able to detect certain degrees of freedom?") Furthermore, the vector **x** describes plainly the degrees of freedom, whereas **f** and **h** assign all measurements to **x**. (Wagner (2003) has compiled the most important versions of **f** and **h**.)

Category	Degrees of freedom considered		Typical sensor types for	
	rotational	translational	**u**	**y**
directional gyro	1 (yaw)	–	gyro	magnetic compass
artificial horizon	2 (pitch, roll)	–	gyro	levelling sensors or accelerometer
attitude and heading reference systems	3 (pitch, roll, yaw)	–	gyro	accelerometer, magnetic compass
classical dead reckoning	–	2 (latitude, longitude)	log / wheel sensor, magnetic compass	GPS receiver
extended dead reckoning	1 (yaw)	2 (latitude, longitude)	log / wheel sensor, gyro	GPS receiver, magnetic compass
INS (inertial platform, strapdown system)	3 (pitch, roll, yaw)	3 (lat., long., altitude)	gyro, accelerometer	GPS receiver, radar, barometer

Table 1: Motional Degrees of Freedom considered in traditional integrated navigation systems.

3. DEVELOPMENT OF INERTIAL NAVIGATION SYSTEMS

In navigation, each vehicle is traditionally being regarded as a solitary rigid body. Assuming the general case of a free spatial motion, it has consequently three rotational and three translational degrees of freedom. An aided INS is therefore the most extensive integrated navigation system (Table 1). Historically, inertial navigation systems emerged during the Second World War, when the flight control for the launch vehicle A4 (the missile V2) required to consider the rocket motion in full (Wrigley, 1977): The determination of the attitude and heading by means of gyros (Table 1) had to be completed by calculating the vehicle velocity and position through numerically integrating accelerometer signals.

In older, early INS, all sensors are attached to a gimballed platform, which is typically kept horizontal and kept orientated to north during the flight. The attitude angles of a vehicle moving "around" the platform correspond therefore to the gimbal angles. As the classical platform alignment is based furthermore on mechanical gyros, it is possible to interpret the gimbals as the output unit of a mechanical computer determining now the complete attitude by suitably integrating the

angular velocity detected through the gyros.
Showing suspensions with one or two gimbals, the gyros of the early inertial navigation systems resemble directional gyros. Considering also the gimbals of the platform itself, these mechanical systems are considerably subject to environmental influences and wearout effects. Although well proven, they require a high level of maintenance; they are costly and have a high weight. Optical gyroscopes (laser gyros, fibre optical gyros) with nearly no moving parts represent here an advantageous alternative (Lawrence, 1993). Their use spread about three decades ago. It was accompanied by the gradual transition from gimballed platforms to strapdown systems: Omitting any articulated suspension, this newer variant of inertial navigation systems has the essential feature that all inertial sensors (gyros and accelerometers) are attached directly to the vehicle. In parallel, the position, the velocity, and the attitude of the vehicle are determined now all by numerically integrating a set of ordinary differential equations. This procedure can be directly attributed to the lower left block of Figure 4. Here, the input **u** is composed of the angular rate vector and the acceleration vector of the vehicle as measured by (at least) three rate gyros and three accelerometers. (These six sensors form the inertial measurement unit IMU of the INS.)
Mathematically, the numerical treatment of $\dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}},\mathbf{u})$ is the solution of an initial value problem. Independently from using a mechanical platform or a digital computer, it includes inevitably the generation of numerical errors, which increase steadily with time. Therefore, an inertial navigation system can only be operated for a restricted period of time unless an additional procedure limits the long-term errors to a reasonable size. This is the task of the aiding.
Interestingly, the historical development of aided inertial systems did not refer to Figure 4 directly. The functional separation between stabilized platforms and early on-board computers as well as the limited digital precision of early microprocessors lead intuitively to a segmentation of the two model blocks in the middle. Figure 5 shows this partition. The upper two model blocks contain the computation of the main part of $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, i.e. the more significant digits. The lower two blocks calculate the less significant digits using basically the Jacobians **F** and **H** of **f** and **h**. As usual done, these second parts of $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ can be interpreted at the same time as the errors $\delta\hat{\mathbf{x}}$ and $\delta\hat{\mathbf{y}}$ caused by the limited precision of the upper model blocks and of the sensors measuring **u**. In case the vector function **f** is nonlinear, as usual in navigation, the avoidance of linearization effects requires from time to time to an additional update of $\hat{\mathbf{x}}$ by $\delta\hat{\mathbf{x}}$ (dotted line in Figure 5).

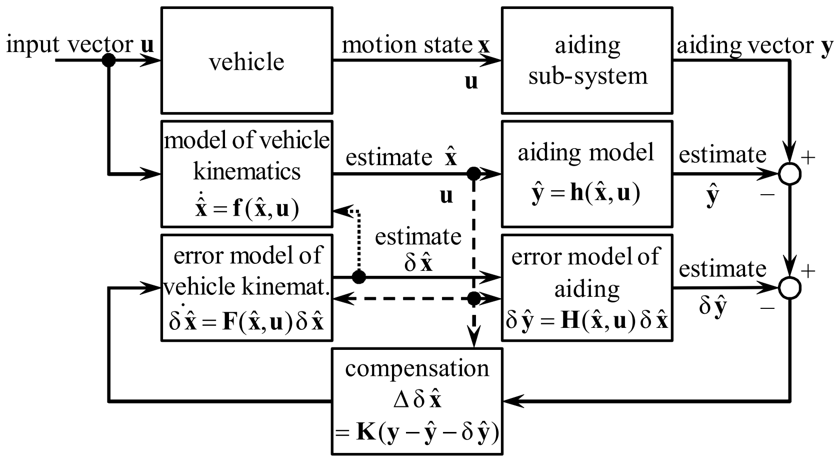

Figure 5: Modified observer principle employed classically for aiding inertial navigation systems.

Considering INS technology during the last two decades, three trends are noteworthy in particular:

- increasingly accurate aiding by satellite navigation receivers (especially GPS devices),
- development of low-cost microelectromechanical gyros and accelerometers (MEMS technology) with moderate accuracy (Maluf, 2000),
- availability of very powerful microprocessors.

Besides supporting the first and the second trend, the last one promotes specially the system integration and allows in particular using the simpler structure of Figure 4 instead of Figure 5. This is important as the utilization of low-cost sensors causes a raised overall error level questioning the linearization approach of Figure 5. Figure 4 offers here an alternative. It is directly designed to handle nonlinear functions **f** and **h** if for example the theory of the Extended Kalman Filter provides the matrix **K**. Indeed, the evaluation of flight tests with a MEMS IMU (see section 4 below) shows improved system stability for the simpler system structure compared to the classical one.
The first point, the good aiding accuracy of GPS, favours also the use of low-cost inertial sensors because it is especially able to dampen the mentioned increase of the system error level. This is the reason why during the last years low-cost systems based on a MEMS IMU and a satellite navigation receiver became more and more interesting for demanding navigation applications.

4. INTEGRATING INERTIAL AND SATELLITE NAVIGATION

Integrated navigation systems based on inertial sensors and GPS combine normally the strapdown technology with a one-antenna satellite navigation receiver. Due to their importance obtained during the last years, there are a number of books describing the details of the system design (e.g. Farrell & Barth, 1999; Jekely, 2001). These publications reflect that customary equipment is still based on the classical structure of Figure 5. Moreover, they show that the schemes of Figure 4 and 5 allow a certain freedom in defining **u**, **x**, and **y**. Wagner (2003) has therefore written a survey comparing several system variants. Concerning accuracy, system stability, and computational effort, one design proved to be especially favourable (see also Figure 6 below). It utilizes, as mentioned above, the simpler structure of Figure 4 as well as the following composition of the vectors **u**, **x**, and **y**:

$$\mathbf{u} = \begin{bmatrix} \text{vehicle acceleration vector} & \mathbf{a}_b \\ \text{vehicle angular rate vector} & \boldsymbol{\omega}_b \end{bmatrix} \quad \begin{matrix} \text{(3 components, vehicle-fixed coordinate system)} \\ \text{(3 components, vehicle-fixed coordinate system)} \end{matrix}$$

$$\mathbf{x} = \begin{bmatrix} \text{position vector} & \mathbf{x}_p \\ \text{velocity vector} & \mathbf{x}_v \\ \text{attitude quaternion} & \mathbf{x}_q \\ \text{biases of all gyros} & \mathbf{x}_\omega \\ \text{biases of all accelerometers} & \mathbf{x}_a \\ \text{GPS receiver clock bias and drift} & \mathbf{x}_c \end{bmatrix} \quad \begin{matrix} \text{(3 comp., earth-centered, earth-fixed coordinate system)} \\ \text{(3 comp., earth-centered, earth-fixed coordinate system)} \\ \text{(4 comp., earth-centered, earth-fixed coordinate system)} \\ \text{(3 elements)} \\ \text{(3 elements)} \\ \text{(2 elements)} \end{matrix}$$

$$\mathbf{y} = \begin{bmatrix} \text{pseudo range, 1}^{\text{st}}\text{ satellite} & y_1 \\ \text{pseudo range rate, 1}^{\text{st}}\text{ satellite} & y_2 \\ \text{pseudo range, 2}^{\text{nd}}\text{ satellite} & y_3 \\ \text{pseudo range rate, 2}^{\text{nd}}\text{ satellite} & y_4 \\ \vdots & \vdots \end{bmatrix} \quad \begin{matrix} \text{(1 element)} \\ \text{(1 element)} \\ \text{(1 element)} \\ \text{(1 element)} \\ \end{matrix}$$

As the associated functions **f** and **h** are repeatedly documented (e.g. Wagner, 2003), it is not essential to repeat them here. Nevertheless, two points concerning **h** are noteworthy. The first one relates to the location of the GPS receiver antenna relative to the IMU. Antenna and IMU are two separate components. They cannot be attached to the vehicle at exactly the same point, and the lever arm between them must be included into **h**. The second point deals also with this position difference. However, it pertains primarily to the stability of the feedback loop of in the lower part of Figure 4 and 5. To get a reliable system performance, it is indispensable that the correction output of the compensation block affects all elements of $\hat{\mathbf{x}}$ (or $\delta\hat{\mathbf{x}}$ respectively). This necessary property is firstly determined by the information introduced through the aiding into the systems. (Especially, it is a well-known property that at least the range measurements for four GPS satellites are required.) On the other hand, it depends also on $\mathbf{f}(\hat{\mathbf{x}})$ causing a time-variable system stability. If in particular a single GPS antenna is used for aiding (Hong et al., 2000), the system stability deteriorates significantly when the vehicle is at rest or moves uniformly. Unfortunately, such conditions occur often on critical occasions like observation periods in airborne photogrammetry or automatic landings in flight guidance. For this, Figure 6 gives an impression. It shows the estimation error variance of the attitude of an aircraft performing a test flight with two approaches for landing and with an aerodrome circuit between them. Being representative for a system destabilisation, the variance increases strongly in phases of straight flights and stabilizes during turns. (Wagner (2003) gives additional details about this flight test and the equipment used.) In addition, the diagram illustrates also the better performance of the system structure of Figure 4.

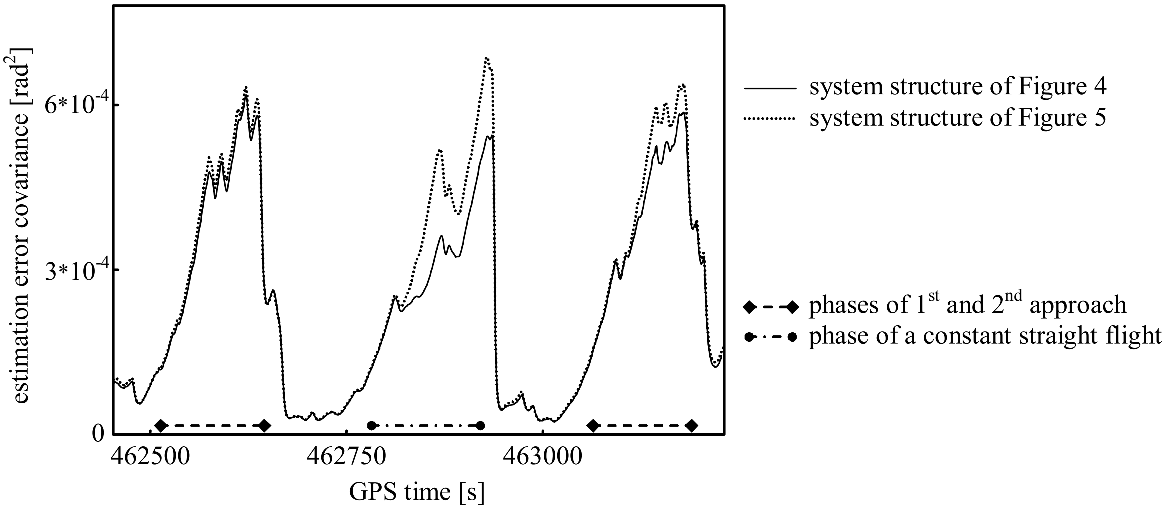

Figure 6: Total attitude estimation error covariance during a flight test.

5. AIDING WITH DISTRIBUTED GPS ANTENNAS

An increasing covariance as illustrated in Figure 6 does not lead necessarily to an extensive reduction of estimation accuracy. Rather, the problem is a reduced stability margin of the feedback loop in Figure 4 and 5, which affects the system reliability as well as the aptitude to handle temporary errors and outages of the sensor signals. A sustainable solution for this inconvenience can be provided by a multi antenna GPS receiver. To exemplify the last statement, Figure 7 shows three IMU-antenna configurations used for a simulated flight test (Wagner, 2003). The ground path of the flight is given in Figure 8. The IMU properties assumed corresponded to a MEMS unit.

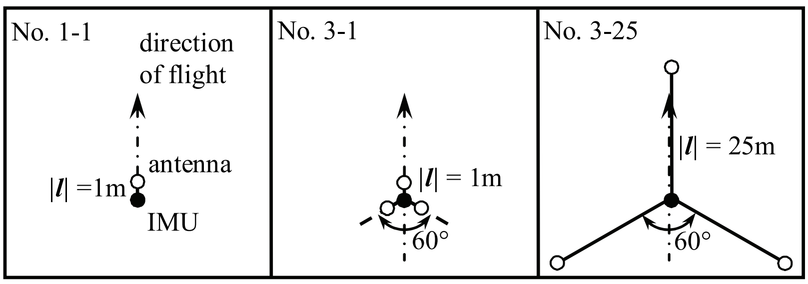

Figure 7: IMU-antenna configurations of a simulated flight test.

Taking a flight time of 30 minutes, the flight track was passed once. Near the crossing point in the centre of Figure 8, the simulated aircraft experienced phases of comparatively uniform motion. These took place at the beginning, in the middle, and the end of the flight. During those periods, the

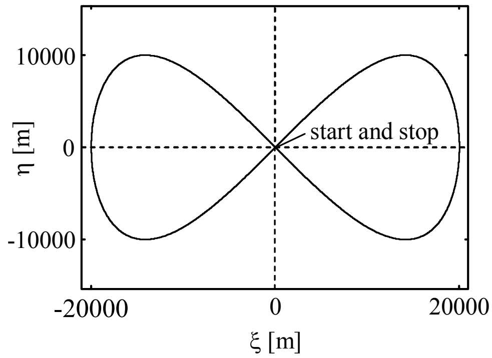

Figure 8: Ground path of a simulated test flight.

estimation error variance of the attitude showed again a significantly high level. Corresponding to Figure 6, Figure 9 reveals clearly this effect. However, it contains furthermore two diagrams, the left one for an aiding of an accuracy of 1 m (corresponding to differential GPS), and the right one for an aiding of an accuracy of 4 cm (GPS carrier phase measurements). Interestingly, the rise of the covariance does disappear neither if the aiding accuracy is improved nor if a small multi antenna array is employed. The only way to remove this undesired effect is the use of a widely distributed multi antenna array. On the other hand, this approach causes problems due to vehicle flexibilities.

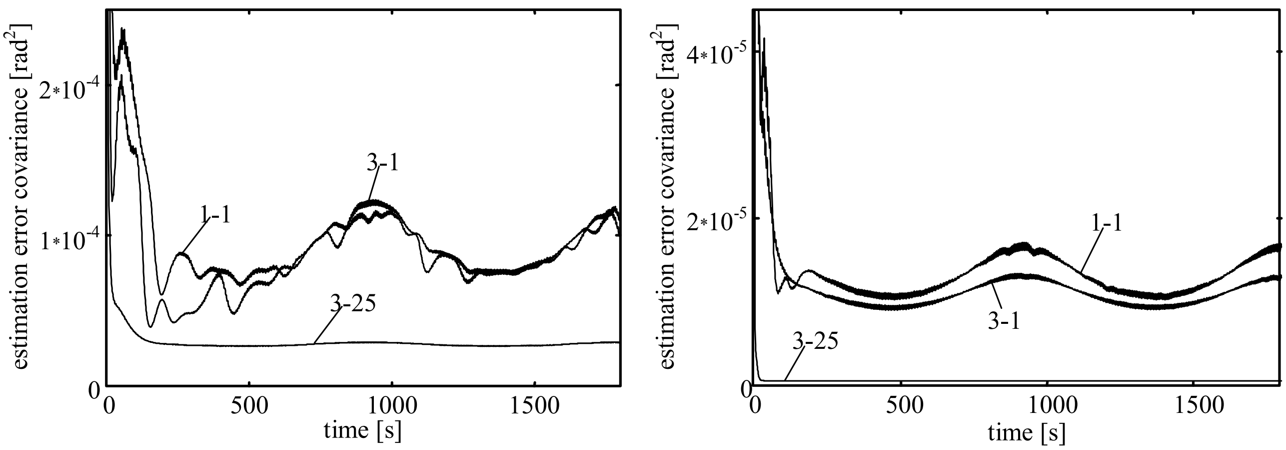

Figure 9: Total attitude estimation error covariance during a simulated flight test;
left: aiding accuracy of 1 m; right: aiding accuracy of 0.04 m.

The classical approach to circumvent the influence of vehicle distortions is to attach all navigation sensor elements close together on a relatively rigid part of the structure. The increasing use of multi antenna systems has nevertheless initiated to consider also relative movements of the aiding components. Besides filtering techniques, which only average structural vibrations, pure GPS procedures exist, which consider explicitly flexibilities, but their sample rate and real time capability is limited. (Wagner (2003) has compiled a corresponding literature survey.) It is therefore better to detect time-varying shapes of vehicles also with inertial sensors by distributing all signal sources explicitly over the structure. To this, Stieler (1999) published a first approach for the corresponding problem regarding robots, which was generalized for arbitrary multibody systems by Wagner (2004). An equivalent theory exists also for elastically deformed vehicles as outlined in brief.

Forming the starting point, Figure 10 shows a fuselage with a distorted wing half attached; the original wing shape is indicated by the dashed line. An aircraft-fixed coordinate system serves for describing the time-variant structural geometry. An IMU with three gyros and three accelerometers is in the origin of the coordinate system. Distributed over the structure, there are satellite navigation antennas at positions *j*. Due to structural deformations, these elements undergo time-variant dis-

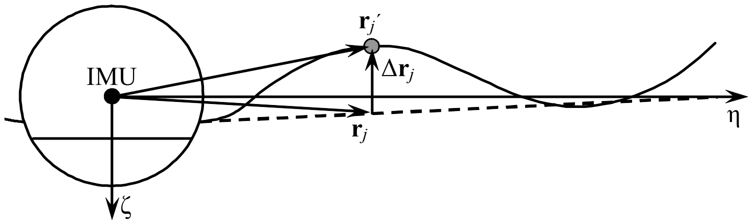

Figure 10: Fuselage cross-section with distorted wing half and peripheral sensors.

placements $\Delta\mathbf{r}_j$ (Figure 10). Each vector $\mathbf{r}_j'$ forming now an antenna lever arm consists accordingly of the constant original part $\mathbf{r}_j$ and the additional part $\Delta\mathbf{r}_j$. To describe the latter one, it is helpful to employ an approximation by the main vibration modes of the vehicle structure. In detail, every mode κ can be expressed individually by a spatial deformation function $\mathbf{s}_\kappa(\mathbf{r})$ being modulated by the time-variant "amplitude" $b_\kappa(t)$:

$$\mathbf{r}_j' = \mathbf{r}_j + \Delta\mathbf{r}_j \approx \mathbf{r}_j + \sum_\kappa b_\kappa(t)\mathbf{s}_\kappa(\mathbf{r}_j)\,.$$

The modal functions $\mathbf{s}_\kappa$ result from a vibration analysis of the structure and shall be taken for being known. The amplitudes b_κ inhere the property of additional, elastic degrees of freedom. They extend the motion state vector $\mathbf{x}$ and have to be estimated by the Kalman filter, too. However, as a single IMU is only able to measure the filter input $\mathbf{u}$ for the motional degrees of freedom of a rigid body, extra inertial sensors become necessary: Additional accelerometers and gyros have to be distributed over the vehicle at certain positions j. They are subject to the local acceleration vector $\mathbf{a}_j$ and the local angular rate vector $\boldsymbol{\omega}_j$ ($\mathbf{a}_b$ and $\boldsymbol{\omega}_b$ are the vectors measured by the IMU, see section 4):

$$\mathbf{a}_j = \mathbf{a}_b + \boldsymbol{\omega}_b\times(\boldsymbol{\omega}_b\times\mathbf{r}_j') + 2\boldsymbol{\omega}_b\times\dot{\mathbf{r}}_j' + \dot{\boldsymbol{\omega}}_b\times\mathbf{r}_j' + \ddot{\mathbf{r}}_j' \quad \text{with} \quad \dot{\mathbf{r}}_j' \approx \sum_\kappa \dot{b}_\kappa(t)\mathbf{s}_\kappa(\mathbf{r}_j)\,, \quad \ddot{\mathbf{r}}_j' \approx \sum_\kappa \ddot{b}_\kappa(t)\mathbf{s}_\kappa(\mathbf{r}_j)\,,$$

$$\boldsymbol{\omega}_j \approx \boldsymbol{\omega}_b + \frac{1}{2}\sum_\kappa \dot{b}_\kappa(t)\,\mathrm{curl}\,\mathbf{s}_\kappa(\mathbf{r}_j)\,.$$

Corresponding to their measurement axes, the sensors detect components of $\mathbf{a}_j$ and $\boldsymbol{\omega}_j$ respectively, which in turn reflect $\dot{b}_\kappa$ and $\ddot{b}_\kappa$. Assuming now a sufficient number of aptly attached inertial sensors, an ordinary differential equation for each b_κ can be derived from the last relations. They are of the types

$$\ddot{b}_\kappa = \sum_j g_j(\mathbf{a}_j - \mathbf{a}_b, \boldsymbol{\omega}_b, \dot{\boldsymbol{\omega}}_b, \mathbf{r}_j, \mathbf{s}_\kappa(\mathbf{r}_j), \dot{b}_\kappa, b_\kappa) \quad \text{or}$$

$$\dot{b}_\kappa = \sum_j \bar{g}_j(\boldsymbol{\omega}_j - \boldsymbol{\omega}_b, \mathrm{curl}\,\mathbf{s}_\kappa(\mathbf{r}_j))$$

with appropriate functions g_j and $\bar{g}_j$. The differential equations altogether form a set completing the vector function $\mathbf{f}$ (Figure 4) for the additional degrees of freedom.

The expansion of the aiding function $\mathbf{h}$ with respect to all b_κ is simple. Instead of using rigid antenna lever arms, the formula for $\mathbf{r}_j'$ has to be employed. Concerning $\mathbf{h}$, it has furthermore to be mentioned that deformations of vehicle structures can also be measured directly using e.g. strain gauges. These additional, simple sensors offer therefore extra aiding possibilities and can help to reduce the number of GPS antennas required. (Further details about this theory as well as an example based on a simulated flight test are to be found in a thesis of the author (Wagner, 2003).)

6. CONCLUSIONS

Integrated navigation systems based on inertial sensors show a considerable variety reaching from simple instruments like the directional gyro with magnetic aiding to extensive motion measurement networks for flexible vehicles. Furthermore, this diversity suggests

- that there is still a significant development potential for aided inertial navigation systems and
- that the technological basis concerning sensors and signal fusion may also vary in the future.

Nevertheless, integrated systems with inertial components as a central element show some general properties that determine their applicability. These characteristics shall finally be ordered as follows:

- Strengths:
  - high resolution with respect to time,
  - good short-term and good long-term accuracy,
  - no jamming of inertial measurements,
  - identification and elimination of poor aiding measurements,
  - adaptable level of accuracy.
- Weaknesses:
  - high complexity with respect to the sensor equipment and the mathematical algorithms,
  - insufficient aiding (e.g. only one GPS antenna) leads to system instability.
- Opportunities:
  - extremely flexible and powerful navigation systems,
  - potential for applications with a high level of automation/autonomy,
  - considerable variety regarding new applications.
- Threats:
  - high-level development work necessary,
  - low-cost systems require a mass market.

Increasingly powerful microprocessors and software tools have the potential to reduce the development costs and development risk for integrated navigation systems. The long-term decrease of the price/performance ratio for inertial and aiding sensors allows furthermore lowering the production costs. Therefore, there is a realistic chance that integrated navigation systems with inertial sensors will become less exclusive but much more common than today.

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